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Math Necessities

for Review and/or Instruction

for

- Learning Support
- College Algebra
- Mathematical Models
- Quantitative Skills
- SAT/ACT review
- GACE review

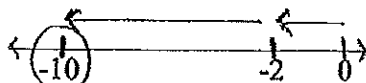
Rules for Operations with Integers

Name _____

I. Addition and subtraction

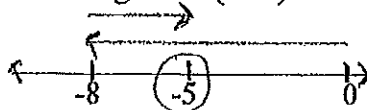
A. $3 + 4 = 7$ Positive + Positive = Positive (Add)

B. $-2 + -8 = -10$



Negative + Negative = Negative (Add)

C. $-8 + 3 = -5$
(Subtract)



To add a positive and a negative, you subtract and take the sign of the larger. (Don't rewrite!!). (Subtract)

D. Subtraction: To subtract a #, you add its opposite. (Rewrite if needed.)
(After rewriting, use your rules for addition.)

$$\begin{array}{r} 7 - 2 \\ 5 \end{array} \quad \text{OR} \quad \begin{array}{r} 7 - 2 \\ 7 + -2 \\ 5 \end{array} \quad \begin{array}{r} 8 - -7 \\ 8 + +7 \\ 15 \end{array}$$

II. Multiplication and Division

A. $3 \times 4 = 12$ Positive x Positive = Positive
Why? 3×4 means $4 + 4 + 4 = 12$

B. $3 \times -2 = -6$ Positive x Negative = Negative
Why? 3×-2 means $-2 + -2 + -2 = -6$

C. $-2 \times -5 = 10$ Negative x Negative = Positive
Why? -2×-5

$$\begin{array}{l} (-)(2)(-)(5) \\ (-)(-)(2)(5) \\ (+)(10) \\ (10) \end{array}$$

A student I tutored a few years ago, once had many difficulties with Trig/Pre-Calc mainly because he didn't know these rules well; he understood the more complex math concepts.

General Guidelines for Solving Equations

Name _____

Goal: To get the variable all by itself on one side of the equation, with only a coefficient of one.

1. If needed, clear fractions and decimals.

Fractions- by multiplying both sides of the equation by the LCD

Decimals- by multiplying both sides of the equation by a power of 10 (10,100,1000, etc.)

2. Simplify both sides (distribute, combine (add or subtract) like terms)

Like terms- terms that are exactly alike except for their numerical coefficients - have exactly the same variables with exactly the same exponents

When you add or subtract like terms, the only thing that changes is the numerical coefficient (the number out front)

3. Add or subtract to get all the variables (letters) on one side

4. Add or subtract to get all the constants (numbers w/o letters) on the other side

5. Multiply or divide to get the coefficient of the variable to be one

-Most of the time, use the inverse operation

(multiplication or division)

-Keep goal in mind; if coefficient is a fraction, multiply by the reciprocal

6. Check

Exponents

Name _____

I. When you multiply like bases you add exponents.

$$m^5 \cdot m^4 = m^9 \quad 3x^2y^4 \cdot 5x^8y^3 = 15x^{10}y^7$$

II. When you divide like bases you subtract exponents.

$$\frac{20x^7p^3}{35x^5p^8} = \frac{4x^2}{7p^5}$$

III. When you raise a power to a power, you multiply exponents.

$$(3x^7p^5)^4 = 3^4x^{28}p^{20} = 81x^{28}p^{20}$$

IV. When combining (adding or subtracting) like terms, exponents do not change; the only thing that changes is the numerical coefficient. And, you can only combine (add or subtract) terms that have exactly the same variables with exactly the same exponents.

$$13a^4c^5 + 5a^4c^2 + 4a^4c^5 = 17a^4c^5 + 5a^4c^2$$

V. Multiplying Polynomials:

A. When multiplying any polynomials together, you can always use the distributive property. (although, you aren't really distributing if you have a monomial x a monomial.)

B. Only for a binomial x a binomial:

1. FOIL may always be used. This is just a method that helps you make sure each term in the first parentheses gets multiplied times each term in the second parentheses. F (First terms in each parentheses) O (Outer-most terms) I (Inner-most terms) L (Last terms in each parentheses). Then combine like terms.
2. Special product patterns: (Note: FOIL or distributive could always be used on any of these, but the next chapter will be easier if you learn the patterns.)

$$(a + b)(a - b)$$

$$a^2 - b^2$$

$$(a + b)^2$$

$$(a + b)(a + b)$$
$$a^2 + 2ab + b^2$$

$$(a - b)^2$$

$$(a - b)(a - b)$$
$$a^2 - 2ab + b^2$$

Radicals **Simplify:**

Name _____

1. $\sqrt{49}$

2. $\sqrt{98}$

3. $\sqrt{25}$

4. $\sqrt{75}$

5. $\sqrt{100}$

6. $\sqrt{900}$

7. $\sqrt{700}$

8. $\sqrt{400}$

9. $\sqrt{4}$

10. $\sqrt{12}$

11. $\sqrt{16}$

12. $\sqrt{32}$

13. $\sqrt{48}$

14. $\sqrt{9}$

15. $\sqrt{36}$

16. $\sqrt{72}$

17. $\sqrt{27}$

18. $\sqrt{45}$

19. $\frac{3 + \sqrt{49}}{2}$

20. $\frac{3 - \sqrt{49}}{2}$

21. $\frac{8 + \sqrt{36}}{2}$

22. $\frac{8 - \sqrt{36}}{2}$

23. $\frac{12 + \sqrt{16}}{4}$

24. $\frac{12 - \sqrt{16}}{4}$

25. $\frac{30 \pm \sqrt{64}}{2}$

Transformations on Parabolas and Absolute Values Name _____

Parabola

Absolute Value

$$y = x^2$$

$$y = |x|$$

$$y = x^2$$

opens up



$$y = -x^2$$

opens down



$$y = |x|$$

opens up



$$y = -|x|$$

opens down



The number in front of the basic function affects how wide or narrow the graph is:

$$y = 5x^2$$

$$y = \frac{1}{2}|x|$$

Will be narrow

Will be wider

A number added or subtracted will affect the vertex location:

$$y = 3(x - 2)^2 + 4$$

$$y = |x + 5| - 2$$

shifts: $\rightarrow 2$ $\uparrow 4$

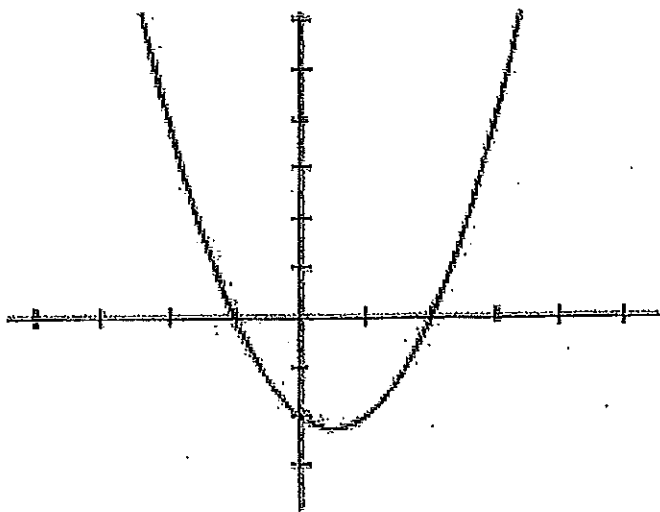
shifts: $\leftarrow 5$ $\downarrow 2$

A #

added or A) inside the basic function, shifts the x in the opposite sign way.
 subtracted: B) outside the basic function, shifts the y in the same sign way.

From: $y = a(x - h)^2 + k$ the vertex is (h,k).

From: $y = ax^2 + bx + c$ $h = \frac{-b}{2a}$, then use h to solve for k.



This parabola is a graph of the equation:

$$y = x^2 - x - 2$$

The zeros are where _____ = zero.

How many zeros are on this parabola? _____

What are the zeros (from the graph)? _____

Two algebraic methods that can be used to find the zeros are:

I. Factoring

$$y = x^2 - x - 2$$

$$y = (x + 1)(x - 2)$$

II. Quadratic Formula

$$y = x^2 - x - 2$$

$$a = 1, b = -1, c = -2$$

Factoring Methods

Name _____

<u>Try when you have:</u>	<u>Method:</u>
Always look for this first.	1. Factor out GCF
2 terms (both squares)	2. Difference of two squares $a^2 - b^2$ factors as $(a + b)(a - b)$
3 terms (2 squares)	3. Perfect square trinomial $a^2 + 2ab + b^2$ $a^2 - 2ab + b^2$ factors as $(a + b)(a + b)$ $(a - b)(a - b)$ $= (a + b)^2$ $= (a - b)^2$
4 terms Try a. when you have 3 perfect squares	4. Factor by grouping a. 1 and 3 or 3 and 1 b. 2 and 2 (involves factoring out GCFs)
2 terms (both cubes)	5. Sum or difference of 2 cubes $a^3 + b^3$ $a^3 - b^3$ factors as $(a + b)(a^2 - ab + b^2)$ $(a - b)(a^2 + ab + b^2)$
3 terms	6. Trial and error When written in descending order, if the 2 nd sign is: a. positive – look for a <i>sum</i> b. negative – look for a <i>difference</i>
Always	7. Remember to factor completely.

Solving Quadratic Equations

Name _____

We will solve quadratic equations by:

1. Factoring:

Get everything on one side = 0 (Important)
and then factor.

Then use the principle of zero products.
(setting each factor = 0, and solving)

2. Taking square roots:

Get the perfect square expression on one side
of the equation by itself.

Take the square root of both sides.

(Don't forget $\pm \sqrt{\quad}$)

Solve and/or simplify further as needed.

3. Completing the square:

Given: $x^2 + bx = \#$

Add $(\frac{1}{2}b)^2$ to both sides to obtain a perfect square on the left.

Rewrite the perfect square trinomial as a binomial squared.

Continue to solve using method 2 above.

****Important:** Make sure the coefficient of the quadratic term is 1 and
there are no constants on the left before you begin this method.

4. Using the quadratic formula:

Get everything on one side = 0. (Important)

Pick a, b, and c out of the form $ax^2 + bx + c = 0$, and substitute into
the quadratic formula. Simplify.

Memorize: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Methods of Factoring
to remember:**

1. Factor out GCF

These next 3 are
easier if you get the
coefficient of the
quadratic term to
be positive 1st:

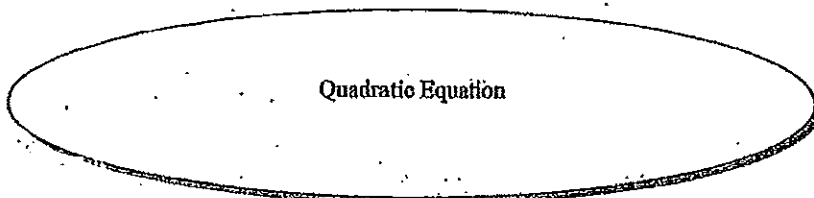
2. Difference of 2
squares

3. Trinomial squares

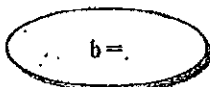
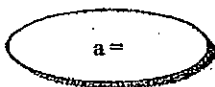
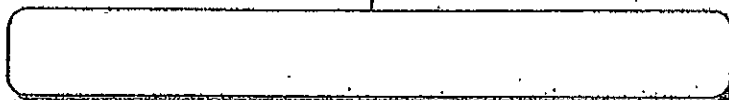
4. Trial & error

Quadratic Formula

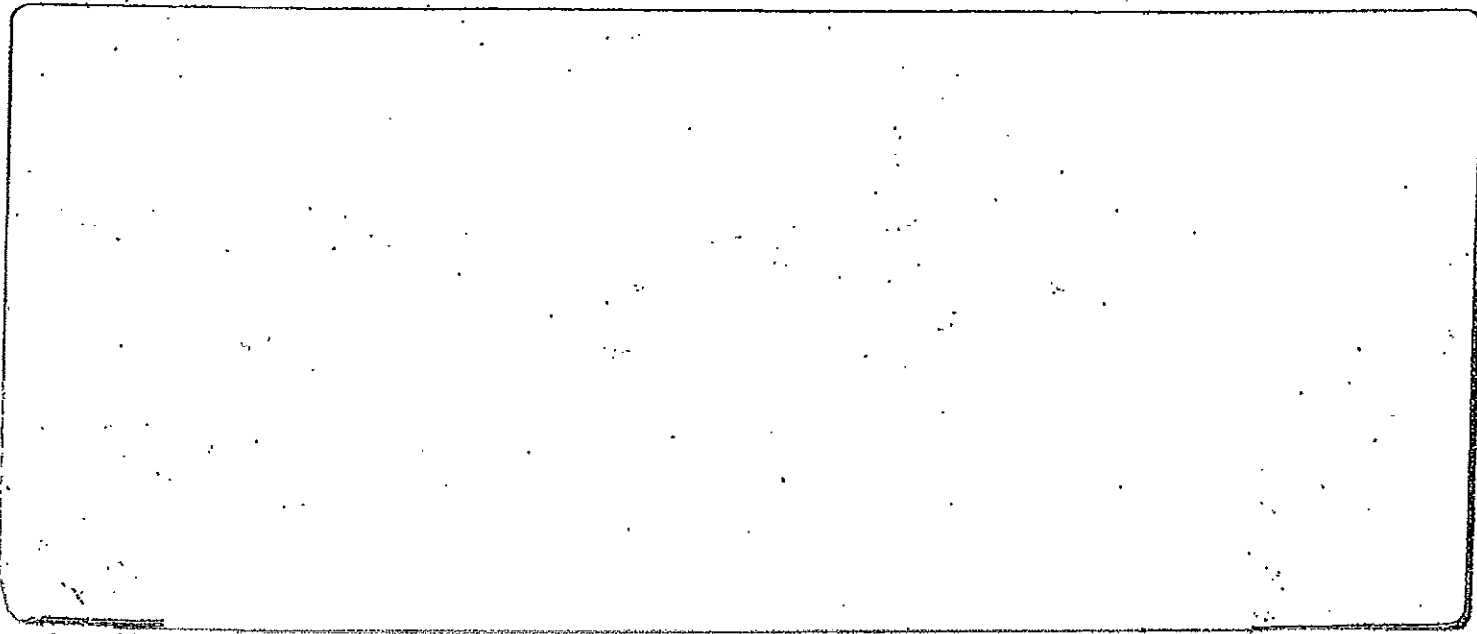
Name _____



set = 0

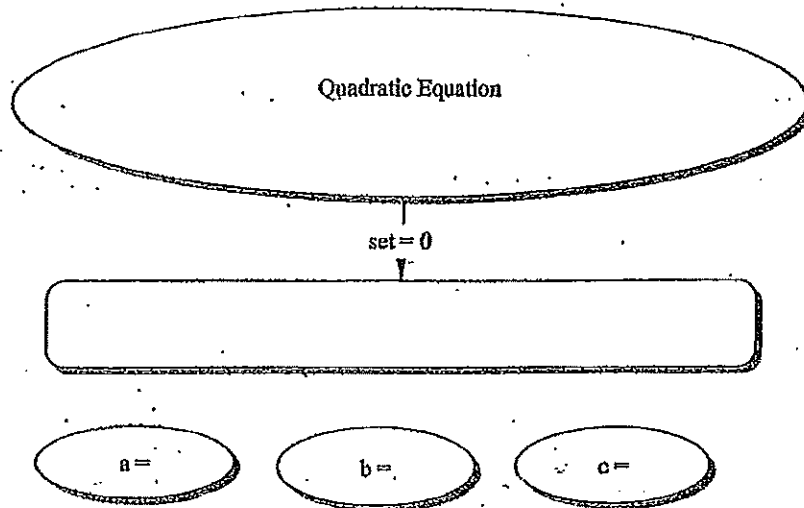


$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

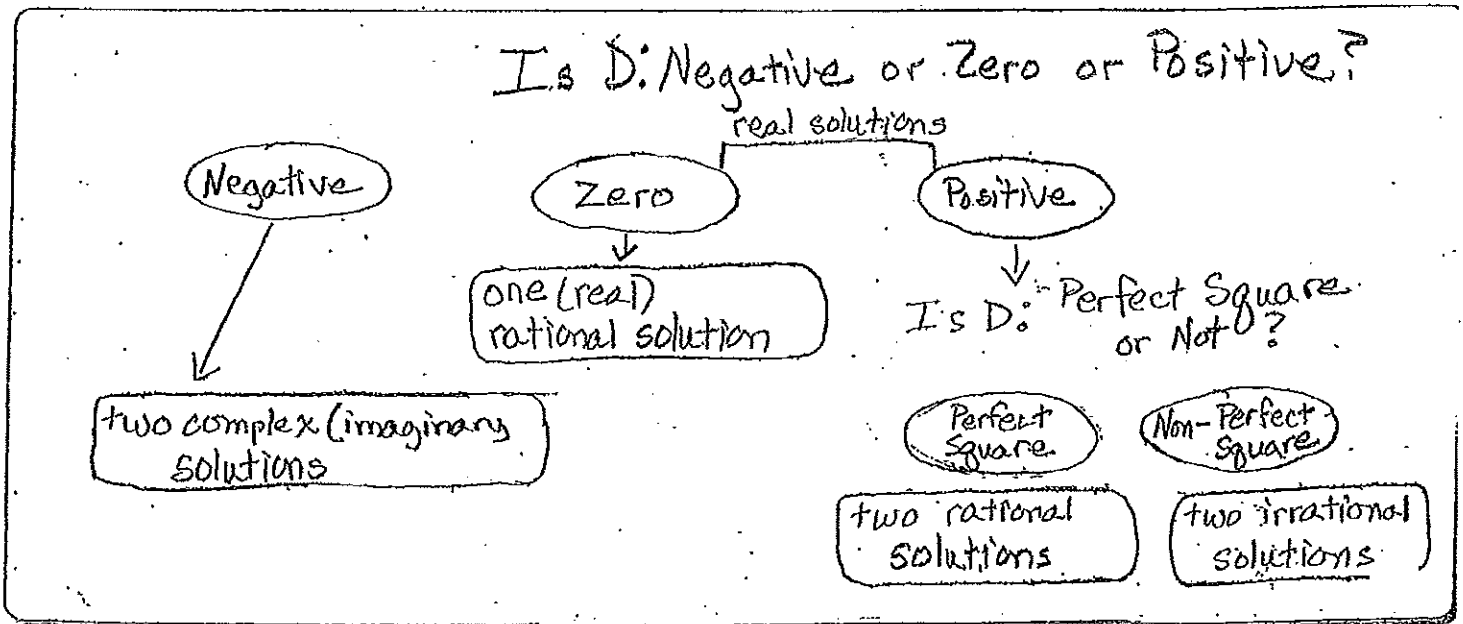


Discriminant/ Determining the Nature of the Roots

Name _____



$$D = b^2 - 4ac$$



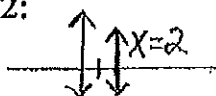
Graphing Lines

Name: _____

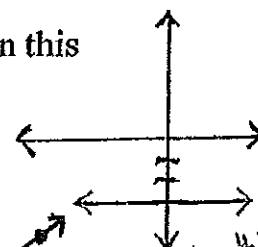
Main methods for graphing lines:

I. Realizing that the equation of the line may tell us that all of the points on the line may have the same x-coordinate (ex: $x = 2$) or the same y-coordinate (ex: $y = -3$).

All the x-coordinates on this vertical line are 2:



All the y-coordinates on this horizontal line are -3:

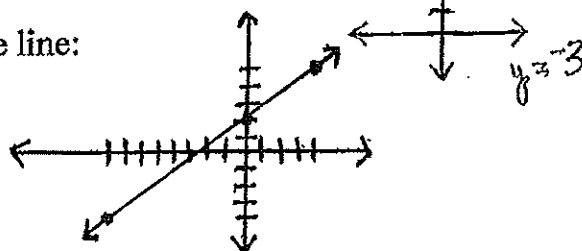


II. Plotting 3 points on the line and drawing the line:

ex: $y = \frac{3}{4}x + 2$

(Note: I chose #'s for x that were divisible by 4 to make the problem easier.)

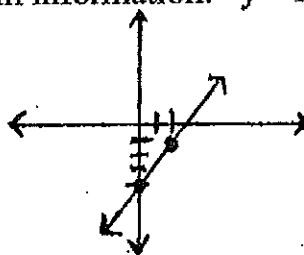
x	y
-8	-4
0	2
4	5



III. Using slope-intercept form information: $y = mx + b$

ex: $y = \frac{3}{2}x - 4$

$m = 3/2$ $b = -4$



(x,y) refers to different points on the line.
 $m = \text{slope} = \frac{\text{rise}}{\text{run}}$
 $b = \text{y-intercept}$
 (the place where the line crosses the y-axis)

- 1st plot -4 on the y-axis
- 2nd using the slope, move to another point on the line from b; plot this point
 $m = \frac{3}{2}$ up 3 or down 3
 2 right 2 left 2
- 3rd draw your line through the 2 points

IV. Plotting the x and y intercepts:

1. Substitute 0 in for x; solve for the y-value. Plot the point.
 Substitute 0 in for y; solve for the x-value. Plot the point.
2. Draw your line through the 2 points.

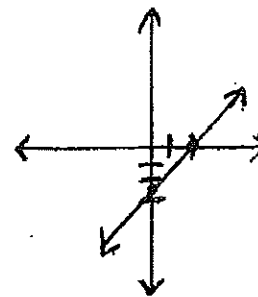
ex: $4x - 3y = 8$

$4x - 0 = 8$ $0 - 3y = 8$

$4x = 8$ $-3y = 8$

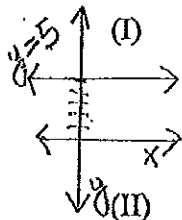
$x = 2$ $y = -8/3$ or $-2 \frac{2}{3}$

x	y	
0	$-2 \frac{2}{3}$	(y-int.)
2	0	(x-int.)



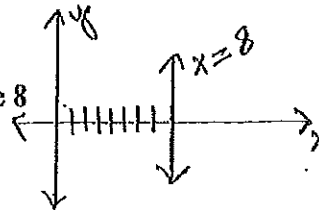
Writing Equations of Lines

Name _____



(I) Horizontal lines
 (-3,5) (7,5)
 all y-coordinates are 5
 $y = 5$

Vertical lines
 (8,-2) (8,3)
 all x-coordinates are 8
 $x = 8$



Using $y = mx + b$ (slope-intercept form)

(A) Given m & b ; just substitute in

$$b = 3, m = 1.8 \quad y = 1.8x + 3$$

(B) Given two points (-3,4) (7,-10)

$$(1) \text{ Find slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10 - 4}{7 - (-3)} = \frac{-14}{10} = \frac{-7}{5}$$

Given \rightarrow the slope and one point start here

(2) Substitute one of the points & the slope into the formula.
 $y = mx + b$
 $-10 = -7/5(7) + b$

(3) Solve for the value of b

$$-10 = -49/5 + b$$

$$-50/5 + 49/5 = b$$

$$-1/5 = b$$

(4) Substitute just m & b into formula; leave x & y to stand for all general points on the line.

$$y = -7/5x - 1/5$$

* Note: If one point has "0" as the x-coordinate, b is the y-coordinate in that point. (i.e.) (0,5) $b=5$

(III) Using $y - y_1 = m(x - x_1)$ (point-slope form)

(A) Given the slope and one point; just substitute the point in for (x_1, y_1) & the slope in for m , then simplify to get in the desired form.

Ex: (3,-5) $m = 7$, put the answer in slope-intercept form

$$y - (-5) = 7(x - 3)$$

$$y + 5 = 7x - 21$$

$$y = 7x - 26$$

(B) Given two points: (7,4) (-8,2)

$$(1) \text{ Find the slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{7 - (-8)} = \frac{2}{7 + 8} = \frac{2}{15}$$

(2) Substitute one of the points in for x_1 and y_1 , and the slope in for m , & simplify to get in the requested form. (ex: standard form)

$$y - 4 = 2/15(x - 7)$$

$$y - 4 = 2/15x - 14/15$$

$$-2/15x + y = -14/15 + 60/15$$

$$-2/15x + y = 46/15$$

$$2x - 15y = -46$$

ok, but below would look better:

(multiplied both sides by -15 to get this)

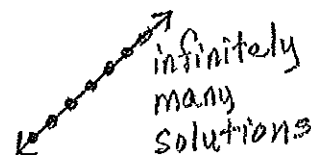
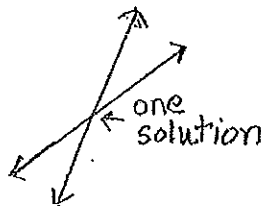
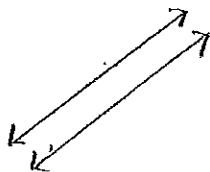
Solving Systems of 2 Equations with 2 Unknowns Name _____

I. Graphing Method

1. Graph one of the lines. Using $y = mx + b$ may be helpful.
2. Graph the other line.
3. The point where the lines intersect is the solution.
4. Write your ordered pair solution.

Possible types of solutions:

- | | | |
|--|--|---|
| A) No solution;
Parallel lines;
Inconsistent
System | B) One solution;
Intersecting lines;
Consistent System | C) Infinitely many solutions;
Equations graph the same line;
Consistent <u>Dependent</u> System |
|--|--|---|



II. Substitution Method

1. Solve for one of the variables in one of the equations.
2. Substitute that expression into the other equation.
3. Solve for the remaining variable.
4. Substitute that new value (into either equation) to solve for the other variable.
5. Write your ordered pair solution.

III. Addition (Elimination) Method

1. Multiply one or both of the equations by a number to obtain opposite coefficients for one of the variables. (This step will not be necessary if opposite coefficients for one of the variables already exist.)
2. Add the 2 equations together (this eliminates one variable).
3. Solve for the remaining variable.
4. Substitute back into one of the equations to solve for the other variable.
5. Write your ordered pair solution.

Solving:

Solve:

$$\begin{aligned} x + 4y - z &= 10 \\ 2x + 5y - 3z &= 7 \\ 8x + y - 2z &= 11 \end{aligned}$$

Write the matrix equation:

$$\begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & -3 \\ 8 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \\ 11 \end{bmatrix}$$

The process overall:

$$\begin{aligned} AX &= B \\ X &= A^{-1} B \end{aligned}$$

Note: Matrix Mult. is NOT commutative; BA^{-1} will NOT give you the correct answer.

I. Enter matrix A, and matrix B into TI-83 plus.

2nd Matrix [2][>] 1 enter 3 enter 3 enter
 1 enter 4 enter -1 enter
 2 enter 5 enter -3 enter
 8 enter 1 enter -2 enter
 2nd Mode (Quit)
 2nd Matrix [2][>] 2 enter 3 enter 1 enter
 10 enter 7 enter 11 enter

II. Solve by multiplying A^{-1} times B

2nd Mode (Quit)
 2nd Matrix 1 [x⁻¹] times 2nd Matrix 2 enter

III. The answer should appear: 2 So $x = 2, y = 3, z = 4$
 3
 4

Inverses

To find inverses:

2nd Mode (Quit)
 2nd Matrix 1 [x⁻¹] enter

To get in fraction form:
 [MATH] Frac enter enter

Determinants:

To find the determinant:

Enter matrix A as above, then:

2nd Mode (Quit)

2nd Matrix [2]

2nd Matrix 1 close parentheses enter

End Behavior (using Highest Degree)

Name _____

When graphing functions whose exponents are positive integers, the end behavior is determined by whether the highest degree exponent is even or odd, and whether the leading coefficient (LC) is positive or negative:

EVEN


ODD


Highest Degree:


2, 4, 6, 8, ...


1, 3, 5, 7, ...

Most Simple Example to Compare to and Remember:

$y = x^2$


$y = -x^2$


$y = x$


$y = -x$


End Behavior (How the graph points at the left and right ends)

If LC is positive: up / up
 If LC is negative: down / down

If LC is positive: down / up
 If LC is negative: up / down

The end behavior is the same at both ends.

The end behavior is different.

Understanding Logarithms

Name _____

log means exponent

log = exponent

then: $\log_2 8$

reads: "log of 8 to the base 2"

means: "What exponent do you raise 2 to, to get 8?" 3

$\log_5 25$ "What exponent do you raise 5 to, to get 25?" 2

If no base is written, then the base is understood to be 10.

$\log 100$ "What exponent do you raise 10 to, to get 100?" 2

$\log 10$ "What exponent do you raise 10 to, to get 10?" 1

$e = 2.718\dots$

\ln means \log_e \ln can be read "natural log"

$\ln e$ "What exponent do you raise e to, to get e ?" 1

The calculator only uses \log with base 10 and \log with base e , so if an expression with other bases can not be easily simplified, you must use the change of base formula.

EXPONENT FACTS

A common sense phrase to remember:

The exponent "goes with" what is immediately beneath it.

$1^2 = 1$	$1^3 = 1$	$(-1)^3 = -1$	$-1^3 = -1$
$2^2 = 4$	$2^3 = 8$	$(-2)^3 = -8$	$-2^3 = -8$
$3^2 = 9$	$3^3 = 27$	$(-3)^3 = -27$	$-3^3 = -27$
$4^2 = 16$	$4^3 = 64$	$(-4)^3 = -64$	$-4^3 = -64$
$5^2 = 25$	$5^3 = 125$	$(-5)^3 = -125$	$-5^3 = -125$
$6^2 = 36$	$6^3 = 216$	$(-6)^3 = -216$	$-6^3 = -216$
$7^2 = 49$	$10^3 = 1000$	$(-10)^3 = -1000$	$-10^3 = -1000$
$8^2 = 64$			
$9^2 = 81$			
$10^2 = 100$		$(-1)^2 = 1$	$-1^2 = -1$
$11^2 = 121$		$(-2)^2 = 4$	$-2^2 = -4$
$12^2 = 144$		$(-3)^2 = 9$	$-3^2 = -9$
$13^2 = 169$		$(-4)^2 = 16$	$-4^2 = -16$
$14^2 = 196$		$(-5)^2 = 25$	$-5^2 = -25$
$15^2 = 225$		$(-6)^2 = 36$	$-6^2 = -36$
$16^2 = 256$		$(-10)^2 = 100$	$-10^2 = -100$
$17^2 = 289$			
$18^2 = 324$		$1^4 = 1$	$1^5 = 1$
$19^2 = 361$		$2^4 = 16$	$2^5 = 32$
$20^2 = 400$		$3^4 = 81$	$3^5 = 243$
$25^2 = 625$		$10^4 = 10000$	$10^5 = 100000$