



Twenty-Sixth Annual Mathematics Tournament
April 15, 2023
Afternoon Component Round #1

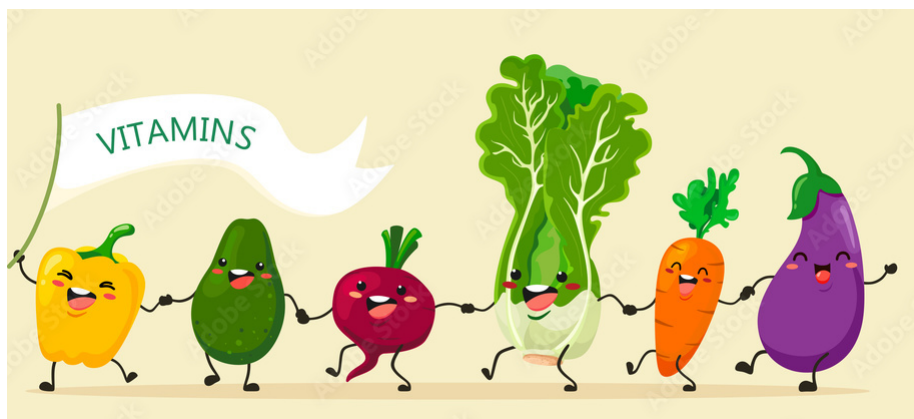
A bag of 30 tulip bulbs contains 10 red tulip bulbs, 10 yellow tulip bulbs, and 10 purple tulip bulbs. If two bulbs are chosen at random, without replacement, what is the probability that one bulb is red and the other yellow? Give an exact answer.





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Afternoon Component Round #2

500 people are surveyed about vitamins B12, C and E. It is found that 52 people took all three vitamins; 150 took B12, 200 took C, 165 took E, 57 took B12 and C, 57 took B12 and E, and 82 took C and E. How many took none of these three vitamins?





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Afternoon Component Round #3

There is a meadow with grass growing on it. 60 cows could graze the grass for 14 days before all the grass is eaten, or 50 cows could graze it for 28 days.

Assuming that each cow eats at the same constant rate, and the grass is growing at a constant rate, what is the maximum number of cows that could graze this meadow and never run out of grass?





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Afternoon Component Round #4

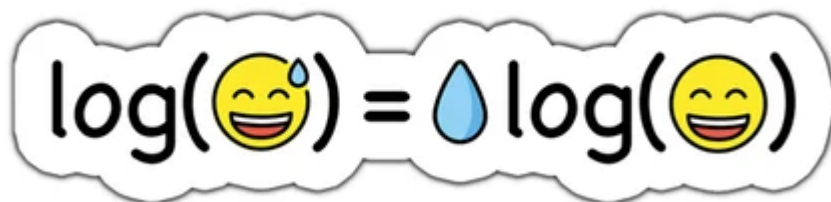
Consider the sequence $3, 3, 2, 1, \dots$. For $n \geq 3$, the n^{th} term is the remainder of the division by 4 of the sum of the previous two terms. Find the 1209^{th} term.





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Suppose $p, q > 0$ and $\log_9(p) = \log_{12}(q) = \log_{16}(p + q)$. Find $\frac{q}{p}$.
Express your answer in a form that involves neither exponentials nor
logarithms.
Give an exact answer.





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For what natural number n is the expression $n^2 - 2n - 8$ equal to a prime number? (Natural numbers are $1, 2, 3, \dots$. Prime numbers are natural numbers greater than 1 whose only factors are 1 and itself.)





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The numbers 2^{2005} and 5^{2005} are written one after the other (in decimal notation). How many digits are written altogether?





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Afternoon Component Round #8

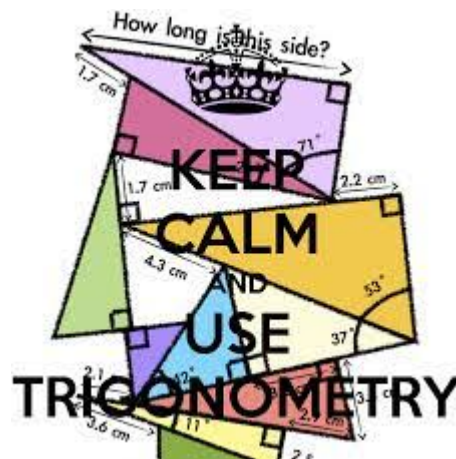
If 5 people are seated in a random manner in a row containing 12 seats, what is the number of ways that a group of three fixed friends always sit adjacently?





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Evaluate $\arccos(k)$, whenever
 $\log_2(\log_4(\log_{\frac{1}{2}}(\log_9(2k)))) = -1$.
Give an exact answer in radians.





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Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region. Give an exact answer.

