1. A survey of 300 tourists visiting Georgia found that the ratio of the number of tourists who visited Stone Mountain to those who visited Atlanta was 2 : 3. Among them, 90 visited both places and 60 visited neither Stone Mountain nor Atlanta. Based on this information, find the number of tourists who visited only Atlanta.

   Let U, S, and A denote the set of all of the tourists who were surveyed and visited Georgia, the set of tourists who were surveyed and visited Stone Mountain, and the set of tourists who were surveyed and visited Atlantarespectively.

   $n(U) = 300, \quad n(S \cap A) = 90, \quad n\left((S \cup A)^c\right) = 60.$

   Let $n(S) = 2x$ and $n(A) = 3x$.

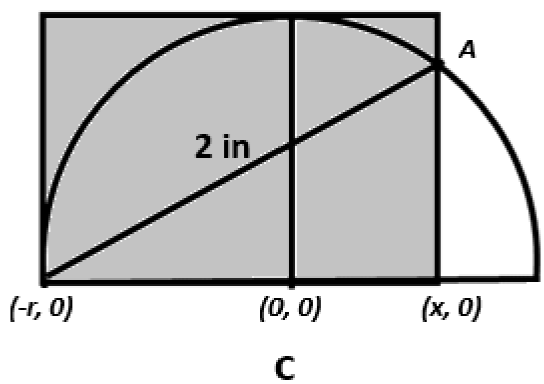
   So, $n(U) = n(A \cup S) + n((A \cup S)^c) = n(S) + n(A) - n(A \cap S) + n((A \cup S)^c)$

   $\Rightarrow 300 = 2x + 3x - 90 + 60 \Rightarrow 300 = 5x - 30 \Rightarrow \dfrac{330}{5} = x \Rightarrow x = 66.$

   So, number of visitors that visited only Atlanta is given by

   $n(A) - n(A \cap S) = 3(66) - 90 = 198 - 90 = 108.$

2. Determine the area of the shaded rectangle, where C is the center of the circle and length of the chord $\overline{AB}$ is 2 inches.



   Let the center and radius of the circle be $(0, 0)$ and r, respectively, and the length of the shaded rectangle be $r + x$.

   Then the point A has coordinates $\left(x, \sqrt{r^2 - x^2}\right)$.

   Note that then $\sqrt{(x - (-r))^2 + \left(\sqrt{r^2 - x^2}\right)^2} = 2$.

   $\Rightarrow (x + r)^2 + r^2 - x^2 = 4.$

$\Rightarrow x^2 + 2xr + r^2 + r^2 - x^2 = 4.$

$\Rightarrow 2xr + 2r^2 = 4.$

$\Rightarrow 2r\,(x + r) = 4.$

$\Rightarrow r\,(x + r) = 2.$

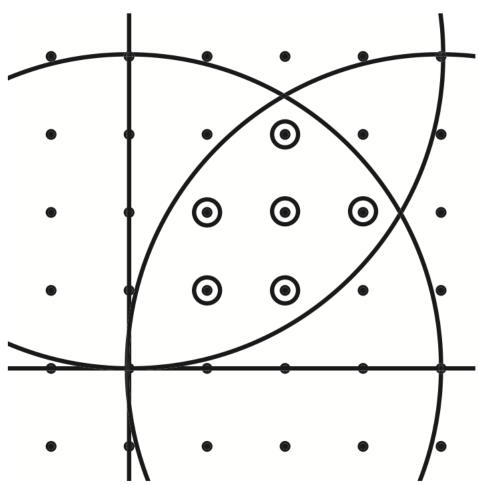
So, the area of the shaded rectangle is $r\,(x + r) = 2$ in^2.

3. How many ordered pairs of integers (a, b) satisfy all of the following inequalities?

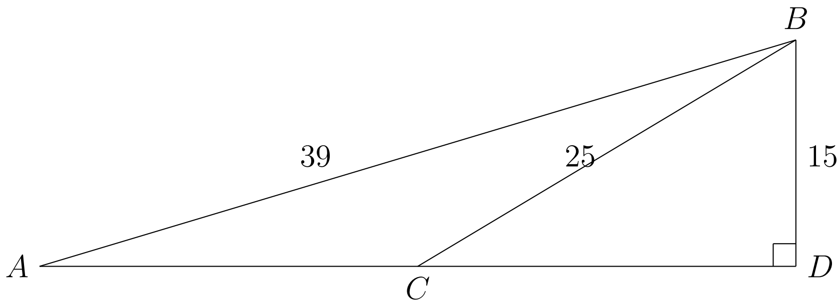
$$a^2 + b^2 < 16$$
$$a^2 + b^2 < 8a$$
$$a^2 + b^2 < 8b$$

Answer is 6.

This is easiest to see by simply graphing the inequalities. They correspond to the (strict) interiors of circles of radius 4 and centers at $(0, 0), (4, 0),$ and $(0, 4)$, respectively. So we can see that there are 6 lattice points in their intersection (circled in the figure).



4. Find the area of the triangle ABC.



Note that length of $CD = \sqrt{25^2 - 15^2} = \sqrt{625 - 225} = \sqrt{400} = 20$ units.

So, length of $AC = \sqrt{39^2 - 15^2} - 20 = \sqrt{1521 - 225} - 20 = \sqrt{1296} - 20 = 36 - 20 = 16$ units.

The semi perimeter of the triangle ABC is $S = \frac{16+25+39}{2} = \frac{80}{2} = 40$ units.

Heron's formula suggests the area of the triangle is $A = \sqrt{S\,(S-16)\,(S-25)\,(S-39)}$

$= \sqrt{40\,(40-16)\,(40-25)\,(40-39)} = \sqrt{40 \cdot 24 \cdot 15 \cdot 1}$

$= \sqrt{2^6 \cdot 3^2 \cdot 5^2} = 2^3 \cdot 3 \cdot 5 = 120$ square units.

5. A florist is buying 200 flowers, consisting of roses, tulips, and carnations for a festival. Roses are \$9 each, tulips are \$7 each, and carnations are \$1 each, and the total budget for flowers is \$1000.

Find the number of each type of flower so that the number of carnations is 60% of the total number of roses and tulips combined.

First calculate the number of carnations. If the number of carnations is 60 percent of the combined number of roses and tulips, say x, then the number of carnations is $\dfrac{60x}{100} = \dfrac{3x}{5}$, that is $\dfrac{\frac{3x}{5}}{\frac{3x}{5} + x} = \dfrac{3x}{8x} = \dfrac{3}{8}$ of the total number of flowers. $\dfrac{3}{8}$ of 200 flowers is 75. There are 75 carnations and these will cost \$75 at \$1 each.

To calculate the number of roses (R) and tulips (T), we can create a system of equations. There are 125 flowers remaining, costing \$925. This creates the following system:

$$R + T = 125 \tag{1}$$

$$9R + 7T = 925 \tag{2}$$

Multiplying the equation (1) by 9 and then subtracting equation (2) from it we get

$$9R + 9T = 1125$$

$$-9R - 7T = -925$$

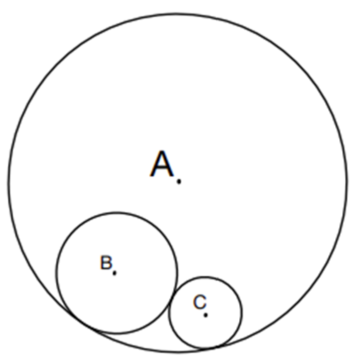
$$2T = 200$$

$$\Rightarrow T = 100$$

Using $T = 100$ in (1) we get $R = 25$.

So, there are 25 roses, 100 tulips, and 75 carnations.

6. We have three mutually tangent circles with centers at points A, B, and C. Assume that the circles are placed as in the picture below:



Assume that the radius of the circle with the center at A is 12. Calculate the perimeter of the triangle with the vertices at the points A, B, and C.

Let r_B be the radius of the circle with the center at B, and r_C be the radius of the circle with the center at C.

Then the distance from A to B is equal to $12 - r_B$.

The distance from A to C is equal to $12 - r_C$.

The distance from B to C is equal to $r_B + r_C$.

The perimeter of the triangle with the vertices at A, B, and C is equal to the sum of the three distances above. So, it is equal to:

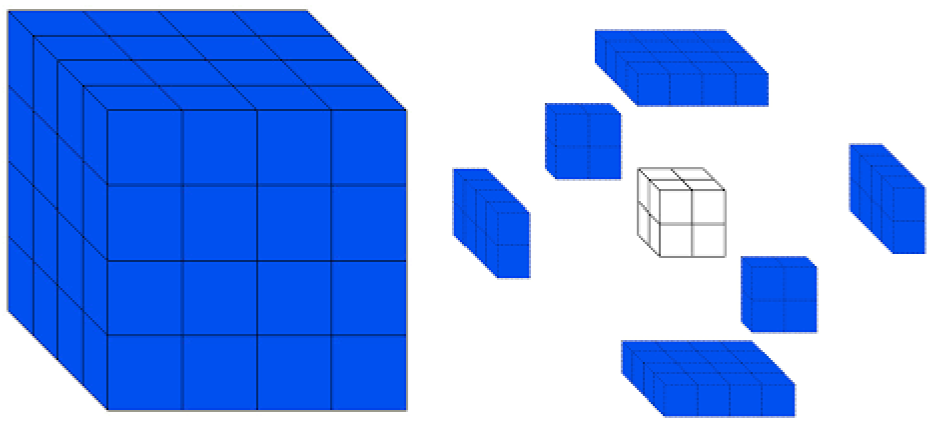
$(12 - r_B) + (12 - r_C) + (r_B + r_C) = 24.$

Final Answer: The perimeter of the triangle is 24.

7. A solid wood cube, measuring 25 centimeters on each of its edges, is painted blue on all six sides. The cube is then cut into smaller cubes, each measuring 1 cm by 1 cm by 1 cm.

   How many of the smaller cubes have either one or two sides painted blue?

   We begin by establishing how many smaller cubes have a side that is part of the outside of the original blue painted cube. Lets begin by orienting the cube as seen below.



   The picture above represents a similar situation with a cube measuring 4 cm on each of its edges.

   If a one cm slice is taken off the top and bottom, each slice could be chopped into 625 cubes (25 cm by 25 cm) that are painted blue on at least one side.

   The remaining part of the cube is now 23 cm high on each face and still 25 cm across.

   Take the front left face and its opposite and cut off a one cm slice. This slice is 25 cm by 23 cm and could be chopped into 575 cubes that are painted blue on at least one side.

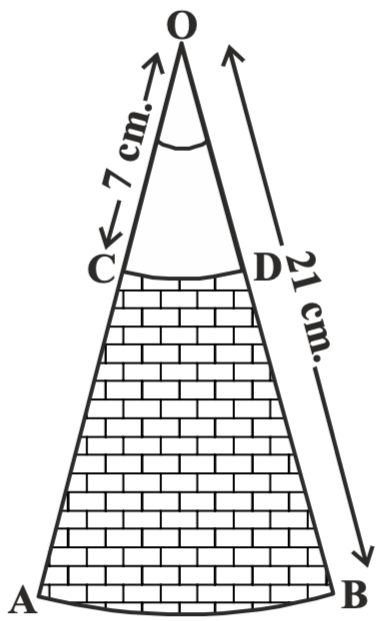
   Now the remaining part of the cube is 23 cm high, 23 cm across one way, and 25 cm across the other way.

   The last two blue faces can be cut to create a one cm slice. This slice is 23 cm by 23 cm and it can be chopped into 529 cubes that are painted blue on at least one side.

This results in cubes with at least one blue painted side.

But the eight cubes at the vertices of the original cube would have three sides painted with blue and thus would be removed from this total to find the number of cubes with one or two painted side, leaving us with 3450 cubes painted blue on one or two sides.

8. AB and CD are arcs of two concentric circles of radii 21 cm and 7 cm, respectively, with center O (See figure). If $\angle AOB = 30°$, find the area of the shaded region (**Use** $\pi = 3.14$). Round your final answer to two decimal places.



Let the area of the sector with radius 21 cm be A_2, and the area of the sector with radius 7 cm be A_1.

Note that $30° = \dfrac{\pi}{6}$.

So, the shaded area is $A_2 - A_1 = \dfrac{\pi\,(21^2 - 7^2)}{12} = \dfrac{392\pi}{12} \approx 102.57 \text{ cm}^2$.

9. If $ab = 100, bc = 200, ac = 300$, find $a + b + c$.

Note that $a \neq 0, b \neq 0, c \neq 0$.

$\dfrac{ab}{bc} = \dfrac{a}{c} = \dfrac{1}{2} \Rightarrow a = \dfrac{c}{2}$.

Hence, $ac = \dfrac{c}{2} \cdot c = \dfrac{c^2}{2} = 300$. Thus, $c = \pm 10\sqrt{6}$.

So, $a = \pm 5\sqrt{6}$. Finally, $b \cdot \left(\pm 10\sqrt{6}\right) = 200$

$\Rightarrow b = \dfrac{200}{\pm 10\sqrt{6}}$

$\Rightarrow b = \pm\dfrac{20\sqrt{6}}{6}$

$\Rightarrow b = \pm\dfrac{10\sqrt{6}}{3}$.

So, $a + b + c = \pm\dfrac{55\sqrt{6}}{3}$.

10. Find all integers x for which the value of the function $f(x) = 1 - \frac{8}{x+1}, x \neq -1$ is a prime number.

For the value of the function $f(x) = 1 - \frac{8}{x+1}$, to be a prime number, we need $1 - \frac{8}{x+1} > 0$ and $\frac{8}{x+1}$ must be an integer. Moreover, $1 - \frac{8}{x+1} > 0$ if $\frac{8}{x+1} < 0$. So, $x + 1 < 0$ and $x < -1$. Also, $x+1$ must be a divisor of 8. Thus, we have the following choices: $x+1 = -8, -4, -2, \text{ or } -1$. So, $x = -9, -5, -3, \text{ or } -2$.

$$\begin{aligned}
f(-9) &= 1 - \frac{8}{-9+1} = 1 - \frac{8}{-8} = 2 \text{ prime} \\
f(-5) &= 1 - \frac{8}{-5+1} = 1 - \frac{8}{-4} = 3 \text{ prime} \\
f(-3) &= 1 - \frac{8}{-3+1} = 1 - \frac{8}{-2} = 5 \text{ prime} \\
f(-2) &= 1 - \frac{8}{-2+1} = 1 - \frac{8}{-1} = 9 \text{ not prime}
\end{aligned}$$

Therefore, $x = -9, -5, \text{ or } -3$.