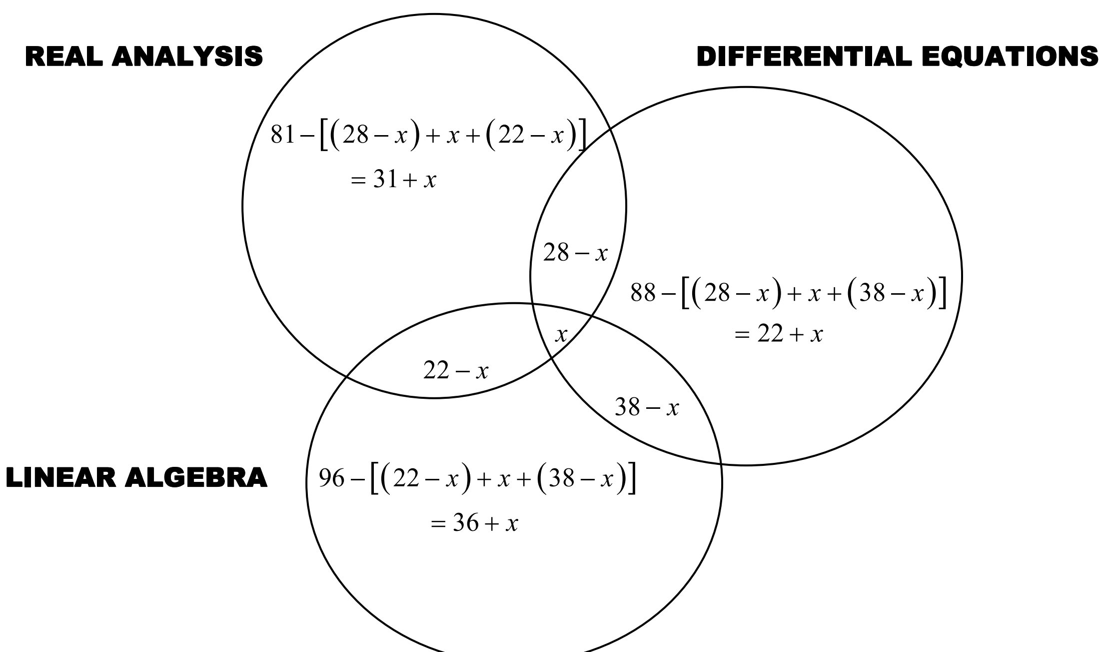
Gainesville College
Tenth Annual Mathematics Tournament
For Two-Year Colleges
April 3, 2004

Solutions for the Afternoon Team Competition

Round 1: Using R = resistance, l = length, and d = diameter, we have $R = \frac{kl}{d^2}$, for some constant k. Solving for k, we get $k = \frac{Rd^2}{l} = \frac{(600)(2)^2}{10} = 240$. Therefore, $R = \frac{kl}{d^2} = \frac{(240)(15)}{(5)^2} = 144\,ohms$.

Round 2: Let x be the number of students enrolled in all three courses. Consider the following diagram:



Adding all numbers and the number of students not enrolled in any of these courses, we obtain the following:

$$20+(31+x)+(28-x)+x+(22-x)+(22+x)+(38-x)+36+x=200$$
$$197+x=200$$
$$x=3$$

There are 3 students enrolled in all three courses.

Round 3: Squaring both sides, subtracting and dividing gives the following:

$$3=\sqrt{1+2\cdot\sqrt{1+3\cdot\sqrt{1+4\cdot\sqrt{x}}}}$$

$$9=1+2\cdot\sqrt{1+3\cdot\sqrt{1+4\cdot\sqrt{x}}}$$

$$8=2\cdot\sqrt{1+3\cdot\sqrt{1+4\cdot\sqrt{x}}}$$

$$4=\sqrt{1+3\cdot\sqrt{1+4\cdot\sqrt{x}}}$$

$$16=1+3\cdot\sqrt{1+4\cdot\sqrt{x}}$$

$$15=3\cdot\sqrt{1+4\cdot\sqrt{x}}$$

$$5=\sqrt{1+4\cdot\sqrt{x}}$$

$$25=1+4\cdot\sqrt{x}$$

$$24=4\cdot\sqrt{x}$$

$$6=\sqrt{x}$$

$$36=x$$

Round 4: We have $(0.04)(2)^{12}\ in=163.84\ in=13.65\overline{3}\ ft$, which is approximately 14 *ft*.

Round 5: Working in the decimal (base 10) notation, $432_{base\ x}$ can be rewritten as $4x^2+3x+2$. Setting this equal to 2004, we get the quadratic equation $4x^2+3x+2=2004$. Solving goes as follows:

$$4x^2+3x+2=2004 \Rightarrow 4x^2+3x-2002=0$$

$$x=\frac{-3\pm\sqrt{(3)^2-4(4)(-2002)}}{2(4)}=\frac{-3\pm\sqrt{32041}}{8}=\frac{-3\pm 179}{8}$$

$$x=22 \text{ or } x=-22\frac{3}{4}$$

Since the negative solution is extraneous, the only possible solution is 22.

Round 6: Begin with the letters. Since all three letters must be the same, the sum of the letters must be worth either 3 points if a vowel is used or 6 points if a consonant is used.

Thus, to arrive at a sum of 32, the vowels must be joined with a series of numbers whose sum is 29, and the consonants must be joined with a series of numbers whose sum is 26.

It is not possible for three single-digit numbers to add up to 29. Thus, if any license plate starts with AAA, EEE, or III, it is not possible to get a sum of 32.

However, it is possible for three single-digit numbers to add up to 26. A combination of two 9's and one 8 will have a sum of 26. There are three possible combinations of two 9's and one 8. These are 899, 989, and 998. Since there are seven consonants in the list above and for each consonant there are three ways to make 26 points, then there are a total of 21 possible license plates with a value of 32 points.

Round 7: George got the chicken tenders, Frank got the whole granola, and Sally got the deluxe hot dog. We know George didn't get the deluxe hot dog, so he could have ordered the whole granola or the chicken tenders. But someone else got the whole granola, so George had to get the chicken tenders. Furthermore, we know it was Frank who got the whole granola, because he is the only boy in the group besides George. So that leaves Sally with the deluxe hot dog.

Round 8: Let x be the score on the final exam. The test average is 79. The homework average is 87.5.

Since it is not clear if the final exam score will need to replace the lowest test score, we will calculate the result in two ways, without the replacement and with the replacement.

1. Score on the final exam without replacement

$$(0.60)(79)+(0.10)(87.5)+0.30x \geq 80 \;\Rightarrow\; 56.15+0.30x \geq 80$$
$$\Rightarrow\; x \geq 79.5$$

Since the final exam score in this case is larger than the lowest test score, the final exam score will replace the lowest test score.

2. Score on the final exam with the replacement

$$\frac{0.60(x+75+90+84)}{4}+(0.10)(87.5)+0.30x \geq 80$$

$$0.15x(x+249)+8.75+0.30x \geq 80 \;\Rightarrow\; x \geq 75.\overline{3} \;\Rightarrow\; x \geq 76$$

Thus, the student must get at least 76 points on the final exam.

Round 9: Either $2x-4=1$ or $2x-4=-1$, so we have $x=\frac{5}{2}$ or $x=\frac{3}{2}$. Adding these, we arrive at the sum of 4.

Round 10: The area of the circular pizza is $\pi \cdot (8)^2 = 64\pi$. If d is the diagonal of the square and s is the length of a side, then $d^2 = s^2 + s^2 = 2s^2 = 2 \cdot 64\pi$. Thus, $d = \sqrt{2 \cdot 64 \cdot \pi}$. To the nearest tenth this is 20.1 *inches.*