Gainesville State College
Twelfth Annual Mathematics Tournament
April 8, 2006

Morning Component

Good morning!

Please do NOT open this booklet until given the signal to begin.

There are 40 multiple choice questions. Answer the questions on the electronic grading form by giving the best answer to each question.

The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, question number 8 will be used again as a tie-breaker.

This test was designed to be a CHALLENGE. It is difficult, and you may not have time to complete all questions. Do not worry if you are unable to answer several of the questions. Instead, we hope that you will obtain satisfaction from those questions which you ARE able to answer.

You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.

Good luck!

Do Not Open Until Signaled.

Gainesville State College – Twelfth Annual Mathematics Tournament

You may write in this test booklet. Only the electronic form will be graded. Correct answers are awarded one point. Incorrect or blank answers are awarded 0 points.

1. Find the slope of the tangent line to the graph of the equation $y = \ln\left|5 + e^{-2x}\right|$ at the point where $x = 0$.

   a) $-\frac{2}{5}$
   b) $-\frac{1}{3}$
   c) $\frac{1}{3}$
   d) $\frac{2}{5}$
   e) none of the above

2. As a rectangle expands, its length *l* is always 10 meters longer than its width *w*. If the width increases at 2 *m/sec*, at what rate is the area of the rectangle increasing when the width is 5 *meters*?

   a) 15 m^2 */sec*
   b) 20 m^2 */sec*
   c) 30 m^2 */sec*
   d) 40 m^2 */sec*
   e) none of the above

3. The height (in *feet*) of the storm surge *t* hours after a hurricane makes landfall is given by $s(t) = 48\left(12 - 4t + t^2\right)^{-1}$. What are the minimum and maximum heights of the surge during the first 6 hours?

   a) 2 *feet* and 6 *feet*
   b) 0.5 *feet* and 8 *feet*
   c) 4 *feet* and 6 *feet*
   d) 2 *feet* and 8 *feet*
   e) none of the above

4. A circular plate of radius 10 *inches* expands as it is heated. Use differentials to approximate the change in the area of the circle when the radius increases by 0.1 *inches*.

a) $\pi\ in^2$
b) $2\pi\ in^2$
c) $4\pi\ in^2$
d) $10\pi\ in^2$
e) none of the above

5. Let $h(x) = f(x) \cdot g(x)$ where $f'(x) = 2x, g'(x) = 3x^2$, $f(2) = 4$, and $g(2) = 10$. Find $h'(x)$.

a) $5x^4 + 4x^3$
b) $5x^4 + 4x^2$
c) $5x^4 + 4x$
d) $5x^4 + 4$
e) none of the above

6. Evaluate: $\lim_{x \to \frac{\pi}{2}} \left[\left(x - \frac{\pi}{2} \right) \cdot \tan x \right]$

a) -2
b) -1
c) 0
d) 1
e) none of the above

7. Evaluate: $\int_0^1 \frac{x^3 + 2x^2 + x + 3}{(x+1)^2} dx$

a) 1
b) 2
c) 3
d) 4
e) none of the above

Reminder
Question 8 will be used again as a tie-breaker, if necessary.

8. A person 6 *feet* tall is walking away from a street light 20 *feet* high at a rate of 7 *ft/sec*. At what rate is the length of the person's shadow increasing?

a) 2 *ft/sec*
b) 3 *ft/sec*
c) 4 *ft/sec*
d) 5 *ft/sec*
e) none of the above

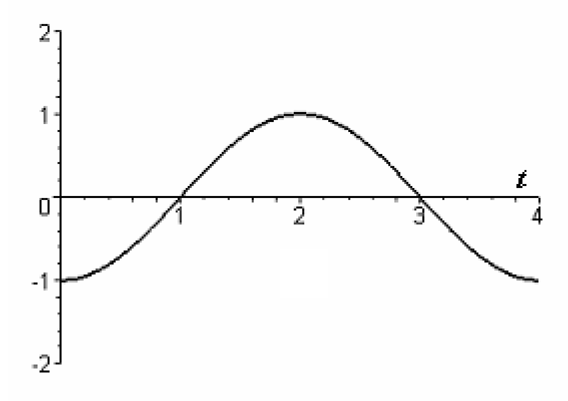
9. Evaluate: $\int_0^\infty xe^{-2x}\,dx$

a) $\frac{1}{5}$
b) $\frac{1}{4}$
c) $\frac{1}{3}$
d) $\frac{1}{2}$
e) none of the above

10. Find an equation of the normal line to the curve $x^2\sqrt{y-2} = y^2 - 3x - 5$ at $(1,3)$.

a) $10x + 11y = 41$
b) $-10x + 11y = 41$
c) $10x - 11y = 41$
d) $11x + 10y = 41$
e) none of these

11. The graph of the function f in the interval $[0,4]$ is given. Identify the x-coordinate of the maximum of the function g on this interval if $g(x)=\int_0^x f(t)\,dt$.



a) $x=1$
b) $x=2$
c) $x=3$
d) $x=4$
e) none of the above

12. What is the limit of the function $g(x)=2+[\![x]\!]+[\![-x]\!]$ as x approaches 2? Recall that the greatest integer function is defined by $[\![x]\!]$ = greatest integer less than or equal to x.

a) 1
b) 2
c) 3
d) 4
e) none of the above

13. Find the x- and y-intercepts of the line that is tangent to the graph of $y=x^3+x^2+x$ at the point $(-1,-1)$.

a) x-intercept is $\left(-\frac{1}{2},0\right)$ and y-intercept is $(0,1)$
b) x-intercept is $\left(\frac{1}{2},0\right)$ and y-intercept is $(0,-1)$
c) x-intercept is $(-1,0)$ and y-intercept is $(0,2)$
d) x- and y-intercepts are $(0,0)$
e) none of the above

14. Suppose that $f(1)=1$ and $f'(1)=2$. Find the value of the derivative of $f(f(f(x)))$ at $x=1$.

a) The value of this derivative is 8.
b) The value of this derivative is 4.
c) The value of this derivative is 2.
d) The value of this derivative is 1.
e) none of the above

15. Suppose that the quadratic function $f(x)=ax^2+bx+c$ is non-negative on the interval $[-1,1]$. Then the area under the graph of f over the interval $[-1,1]$ is given by the formula

a) $A=\frac{1}{2}\left[f(-1)+2f(0)+f(1)\right]$
b) $A=f\left(-\frac{1}{2}\right)+f\left(\frac{1}{2}\right)$
c) $A=\frac{1}{3}\left[f(-1)+4f(0)+f(1)\right]$
d) $A=f(-1)+f(1)$
e) none of the above

16. How many inflection points does the graph of the function $f(x)=x^2+x-\frac{8}{x}$ have?

a) 3
b) 2
c) 1
d) 0
e) none of the above

17. Choose the value of x below that lies in an interval on which the function $f(x)=x^4-12x^3+8$ is both increasing and concave up.

a) -2
b) 3
c) 8
d) 13
e) none of the above

18. What type of relative extrema does the function $f(x) = x^{\frac{2}{3}} e^{-3x^2}$ have?

a) one relative maximum point, no relative minimum point
b) one relative maximum point, two relative minima points
c) two relative maxima points, no relative minimum point
d) two relative maxima points, one relative minimum point
e) none of the above

19. Evaluate: $\lim_{x \to 0^+} \frac{e^{x^2} - e^x + x}{1 - \cos(2x)}$

a) $\frac{1}{4}$
b) $\frac{1}{2}$
c) 1
d) ∞
e) none of the above

20. How many of the following derivatives are correct (on their domains)?

I. $\frac{d}{dx} \ln|\sec x| = \tan x$
II. $\frac{d}{dx} \ln(x + e^x) = 1 + \frac{1}{x}$
III. $\frac{d}{dx} x^{\ln x} = (\ln x) x^{\ln(x)-1}$

a) 0
b) 1
c) 2
d) 3
e) none of the above

21. If $f(x) = x|x|$, then $f'(0) =$

a) 0
b) 1
c) -1
d) does not exist
e) none of the above

22. If $f(x)$ is differentiable at a, what is $\lim_{h\to 0}\dfrac{f(a+h^2)-f(a)}{h}$?

a) 0
b) $f(a)$
c) $f'(a)$
d) $f''(a)$
e) none of the above

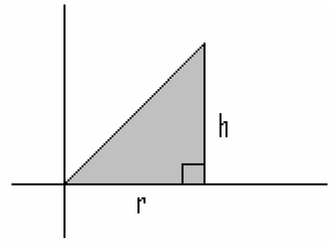
23. For each number c, the function $f_c(x)=x^2-2cx+c$ has a minimum value. What is the maximum of those minimum values?

a) 0
b) $\dfrac{1}{4}$
c) $\dfrac{1}{2}$
d) 1
e) none of the above

24. Which of the following integrals is equal to $\int_{-2}^{2}(4-x^2)\,dx$?

a) $\int_{0}^{4}\sqrt{4-y}\,dy$
b) $\int_{0}^{4}2\sqrt{4-y}\,dy$
c) $\int_{0}^{4}(4-y^2)\,dy$
d) $\int_{-2}^{2}2\sqrt{4-y}\,dy$
e) none of the above

25. If the region shown is rotated around the y-axis, the resulting volume is



a) $\frac{1}{3}\pi r^2 h$

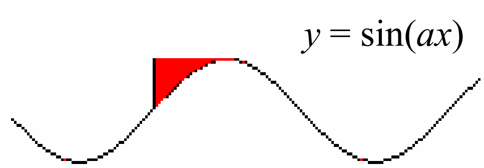
b) $\frac{2}{3}\pi r^2 h$

c) $\frac{1}{3}\pi r h^2$

d) $\frac{2}{3}\pi r h^2$

e) none of the above

26. For what value of $a > 0$ is the area of the shaded region equal to 1?



a) $\frac{\pi}{4}$

b) 1

c) $\frac{\pi}{2}$

d) $\frac{\pi}{2} - 1$

e) none of the above

27. Evaluate: $\lim_{x \to 0} \frac{2^x - 1}{x}$

a) 0

b) 1

c) $\ln(2)$

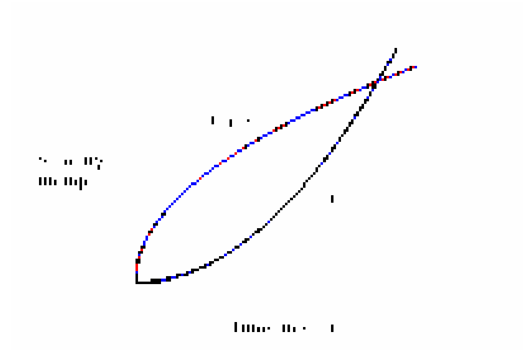
d) does not exist

e) none of the above

28. Let $[\![x]\!]$ denote the greatest integer function, that is $[\![x]\!]=$ greatest integer less than or equal to x. What is $\int_0^{10} [\![x]\!]\,dx$?

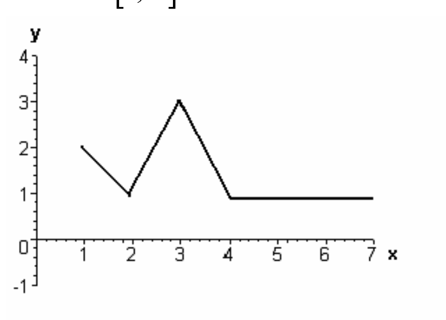
a) 10
b) 45
c) 55
d) $[\![x]\!]$ is not integrable
e) none of the above

29. Two cars starting from rest (at time equal to 0) accelerate to 60 *mph* in 30 *seconds*. The velocity of each car is shown in the figure. The cars start at the same position and are going in the same direction. At the end of the 30-*second* interval:



a) Car 1 is ahead of Car 2.
b) Both cars are side by side.
c) Car 2 is ahead of Car 1.
d) There is not enough information to determine the cars' positions.
e) none of the above

30. The graph of f is given in the figure. Determine the average value of f on the interval $[1, 7]$.



a) $\frac{17}{12}$
b) $\frac{17}{2}$
c) $\frac{11}{6}$
d) $\frac{3}{2}$
e) none of the above

31. Find the derivative of $f(x)=x^e e^x$.

a) $x^{e+1}e^{x-1}$
b) $x^e e^x + x^{e+1}e^{x+1}$
c) $x^e e^x + x^{e-1}e^{x-1}$
d) $x^e e^x + x^{e-1}e^{x+1}$
e) none of the above

32. Evaluate: $\int_0^{2\pi} 3^{\sin^2 x} 3^{\cos^2 x}\, dx$

a) $3^{2\pi}$
b) 2π
c) 6π
d) undefined
e) none of the above

33. Let the function *f* be differentiable such that $f(0)=0$, $f(5)=2$ and $\int_0^5 f(x)\,dx=8$. Then $\int_0^5 f(x)f'(x)\,dx$ is equal to

a) -1
b) 0
c) 1
d) 2
e) none of the above

34. Find the derivative of $f(x)=\dfrac{x^x}{x^5}$.

a) $(x-5)x^{x-6}$
b) $5x^x \ln x$
c) $\dfrac{x^x}{x^6}(x\ln x - x + 5)$
d) $\dfrac{x^x}{x^6}(x\ln x + x - 5)$
e) none of the above

35. Evaluate: $\lim_{x \to 3^+} \dfrac{\ln x^3}{x-3}$

a) ∞
b) e^3
c) 0
d) 1
e) none of the above

36. When $y^3 + y = x$ then $\dfrac{d^2y}{dx^2}$ is equal to

a) $\dfrac{-6y}{\left(3x^2+1\right)^3}$
b) $\dfrac{-6x}{\left(3x^2+1\right)^3}$
c) $\dfrac{-6y}{\left(3y^2+1\right)^3}$
d) $\dfrac{-6x}{\left(3y^2+1\right)^3}$
e) none of the above

37. If $f(x) = g(1-x)$, and $\int_0^1 f(x)\,dx = 2$, then $\int_0^1 g(x)\,dx =$

a) -2
b) -1
c) 0
d) 2
e) none of the above

38. If p and q are positive numbers and $\int_0^p f(x)\,dx = 10$, for which of the following values of q must $\int_0^q 2x\,f\left(x^2\right)dx = 10$?

a) $q = p$
b) $q = p^2$
c) $q = \sqrt{p}$
d) $q = \dfrac{p}{2}$
e) none of the above

39. Let ***R*** be the region in the first quadrant bounded by $x = 0$, $y = 0$, and $y = 1 - x^2$.
Let ***A*** be the volume of the solid obtained by rotating this region around the x-axis.
Let ***B*** be the volume of the solid obtained by rotating this region around the y-axis.
Let ***C*** be the volume of the solid obtained by rotating this region around the line $x = 1$.

Which is largest?

a) A
b) B
c) C
d) $A = B = C$
e) none of the above

40. If $f(x)$ is differentiable and its derivative is everywhere continuous, then $\lim\limits_{h \to 0} \dfrac{f(a+3h) - f(a)}{h} =$

a) $3f'(a)$
b) $f'(a)$
c) $f'(3a)$
d) $\dfrac{1}{3}f'(a)$
e) none of the above