



***Gainesville State College
Fifteenth Annual Mathematics Tournament
April 4, 2009***

Morning Component

Good morning!

Please do NOT open this booklet until given the signal to begin.

There are 40 multiple choice questions. Answer the questions on the electronic grading form by giving the best answer to each question.

The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, question number 35 will be used again as a tie-breaker.

This test was designed to be a CHALLENGE. It is difficult, and you may not have time to complete all questions. Do not worry if you are unable to answer several of the questions. Instead, we hope that you will obtain satisfaction from those questions which you ARE able to answer.

You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.

Good luck!

Do Not Open Until Signaled.

***Fifteenth Annual Gainesville State College
Mathematics Tournament***

You may write in this test booklet. Only the electronic form will be graded. Correct answers are awarded one point. Incorrect or blank answers are awarded 0 points.

1. At the point $(0, 0)$ the graph of $f(x) = |x|$
 - a) has a tangent line at $y = 0$
 - b) has no tangent line
 - c) has infinitely many tangent lines
 - d) has two tangent lines $y = -x$ and $y = x$
 - e) none of the above

2. Calculate the derivative $\frac{df}{dx}$ of the function $f(x) = \int e^{x^2} \cos x \, dx$.
 - a) $e^{x^2} \sin x$
 - b) $e^{x^2} \cos x$
 - c) $x e^{x^2} \sin x$
 - d) $x e^{x^2} \cos x$
 - e) none of the above

3. A designer wants to introduce a new line of bookcases. He wants to make at least 100 bookcases, but not more than 2000 of them. He predicts the cost of producing x bookcases is $C(x)$. Assume that $C(x)$ is a differentiable function. Which of the following must he do to find the minimum average cost, $c(x) = \frac{C(x)}{x}$?

- (I) Find the points where $c'(x) = 0$ and evaluate $c(x)$ there.
(II) Compute $c''(x)$ to check which of the critical points in (I) are local maxima.
(III) Check the values of c at the endpoints of its domain.

- a) I only
b) I and II only
c) I, II and III
d) I and III only
e) none of the above

4. If $|x|$ is very large, then the graph of $f(x) = \frac{2x^3 - 3x^2 + x + 5}{x^2 - x + 1}$ is approaching the line

- a) $y = 2x - 5$
b) $y = 2x + 1$
c) $y = 2x - 1$
d) $y = 2x + 5$
e) none of the above

5. Integrate: $\int \frac{dx}{2x^2 - 5x + 2}$

- a) $\ln|x - 2| + \ln|2x - 1| + C$
b) $\frac{1}{3} \ln|x - 2| - \frac{1}{3} \ln|2x - 1| + C$
c) $\frac{1}{2} \ln|x - 2| - \frac{1}{2} \ln|2x - 1| + C$
d) $\frac{1}{3} \ln|2x - 1| - \frac{1}{3} \ln|x - 2| + C$
e) none of the above

6. A particle is moving in the first quadrant downward on the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$. It leaves the hyperbolic path at the point $(5, 6)$ and continues along a straight line. At what point does the particle cross the x -axis?

- a) $\left(\frac{16}{5}, 0\right)$
- b) $(3, 0)$
- c) $\left(\frac{9}{2}, 0\right)$
- d) $\left(\frac{5}{2}, 0\right)$
- e) none of the above

7. Which statement about the graph of $f(x) = (x + 2)(x - 1)^2$ is **not** true?

- a) f has a relative minimum at $(1, 0)$
- b) f has an intercept at $(1, 0)$
- c) f has a relative maximum at $(-1, 4)$
- d) f has a point of inflection at $(1, 0)$
- e) none of the above

8. Determine the interval on which the function $f(x) = \sqrt{x^2}$ satisfies the assumptions of Rolle's Theorem.

- a) $[0, 1]$
- b) $[-1, 1]$
- c) $[-1, 0]$
- d) $[-2, 3]$
- e) none of the above

9. Calculate the derivative of the function $f(x) = x^{(e^x)}$.

- a) $e^x x^{(e^x-1)}$
- b) $x^{(e^x)} \left[e^x \ln x + \frac{e^x}{x} \right]$
- c) $x^{(e^x-1)} e^x \left[\frac{1}{x} + \ln x \right]$
- d) $e^x x^{(e^x-1)} \ln x$
- e) none of the above

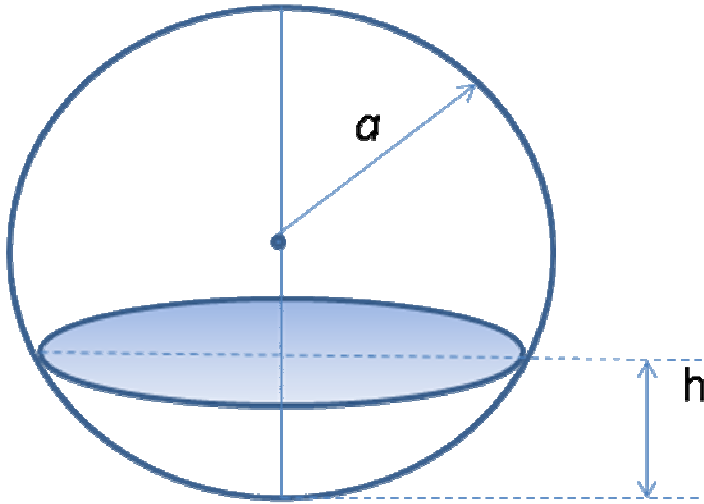
10. Evaluate: $\int_{-7}^7 \tan(x^5 - x^3) dx$

- a) 17
- b) 0
- c) 12
- d) undefined
- e) none of the above

11. Let $f(x) = \sin^{-1}\left(\frac{x-1}{x+1}\right)$, $g(x) = 2 \tan^{-1} \sqrt{x}$, both being defined for $x \geq 0$. Then the following statement is correct.

- a) $f(x) = g(x) + c$, for some constant c
- b) $\int f(x) dx = \int g(x) dx$
- c) The intersection of the domains of the two functions is the set $\{x \mid x < 0\}$
- d) $\frac{d}{dx} \int_1^x f(t) dt = \frac{d}{dx} \int_1^x g(t) dt$
- e) none of the above

12. A hemispherical bowl of radius a contains water to depth h . Find the volume of the water in the bowl.



- a) $\frac{2}{3}\pi a^2 h$
- b) $\frac{2}{3}\pi h^2 \left(h + \frac{a}{3} \right)$
- c) $\frac{2}{3}\pi h^2 \left(a - \frac{h}{3} \right)$
- d) $\pi h^2 \left(a - \frac{h}{3} \right)$
- e) none of the above
13. The radius of a right circular cylinder is increasing at a rate of 0.1 cm/min, and the height is decreasing at a rate of 0.2 cm/min. What is the rate of change of the volume of the cylinder, in cm^3/min , when the radius is 2 cm and the height is 3 cm?

- a) $\frac{2\pi}{5}$
- b) $\frac{5\pi}{2}$
- c) $\frac{\sqrt{2}\pi}{5}$
- d) $\frac{\pi}{3}$
- e) none of the above

14. If f and g are both differentiable and $h = f \circ g$, $h'(2)$ equals

- a) $f'(2) \circ g'(2)$
- b) $f'(2)g'(2)$
- c) $f'(g(x))g'(2)$
- d) $f'(g(2))g'(2)$
- e) none of the above

15. Let $f(x) = x|x|$. Find $f'(0)$.

- a) 0
- b) 1
- c) -1
- d) does not exist
- e) none of the above

16. Evaluate: $\lim_{x \rightarrow 0} \frac{1 - e^{-x}}{\sin x}$

- a) ∞
- b) 1
- c) $\sqrt{3}$
- d) $e^{-\pi}$
- e) none of the above

17. A cylindrical, metal can is to be made to hold 1L of oil. Assume the top, the bottom, and the sides of the can have the same thickness. The dimensions in centimeters that will minimize the cost of the metal to manufacture the can are:

- a) $r = 500^{1/3} \pi^{-1/3}$, $h = 2r$
- b) $r = 500\pi^{-1}$, $h = 2r$
- c) $r = \frac{1}{\pi}$, $h = \frac{r}{2}$
- d) $r = \frac{\sqrt{500}}{\pi}$, $h = 2r$
- e) none of the above

18. Evaluate: $\int_0^{-1} \frac{1}{\sqrt{x+1} + \sqrt{x+1}} dx$

a) $-2\sqrt{2} + \ln(3 + 2\sqrt{2})$

b) $\frac{1}{-2\sqrt{2}} + \ln(3 + 2\sqrt{2})$

c) $-2\sqrt{2} + \ln(2 + 3\sqrt{3})$

d) $\frac{1}{-2\sqrt{2}} + \ln(2 + 3\sqrt{3})$

e) none of the above

19. Find the length of the curve $x = \frac{y^4}{4} + \frac{1}{8y^2}$ from $y = 1$ to $y = 2$.

a) $\frac{125}{32}$

b) $\frac{117}{32}$

c) $\frac{123}{32}$

d) $\frac{121}{32}$

e) none of the above

20. Suppose f and g are differentiable, and that $f(x) = \frac{x^3}{g(x)}$. If $g(4) = 4$ and $g'(4) = 2$, then find $f'(4)$.

a) -16

b) 4

c) 8

d) 0

e) none of the above

21. Evaluate: $\int_0^1 \sqrt{\frac{1+x}{1-x}} dx$

- a) $\pi - 1$
- b) $\frac{\pi}{2} - 1$
- c) $\frac{\pi}{2} + 1$
- d) $\frac{\pi}{4} + 1$
- e) none of the above

22. If $f(x) = \ln|Cx|$, for $C \neq 0$, then $f'(x) =$

- a) $\frac{1}{|x|}$
- b) $\frac{1}{|Cx|}$
- c) $\frac{1}{x}$
- d) $\frac{1}{Cx}$
- e) none of the above

23. If a trigonometric substitution in terms of a secant function in the variable θ is used to

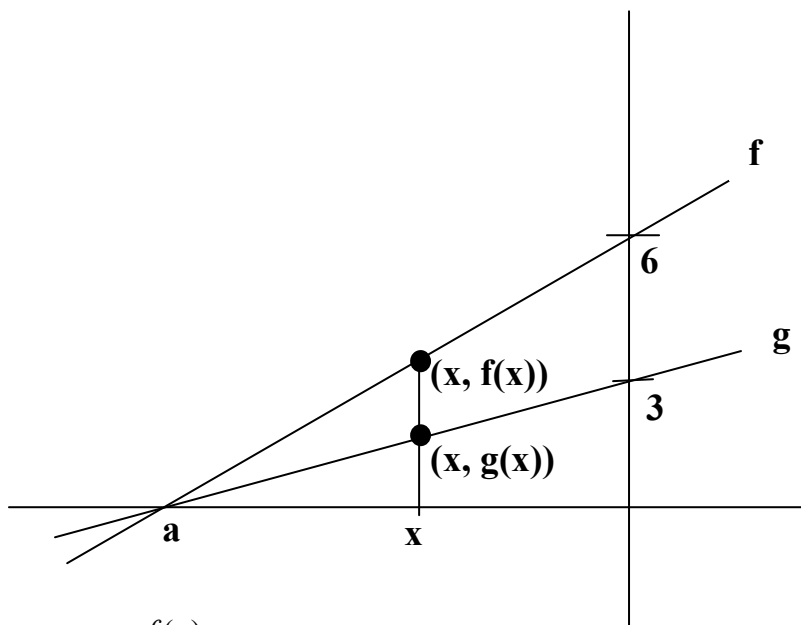
solve $\int_{\frac{5}{2}}^{\frac{5}{\sqrt{3}}} \sqrt{4x^2 - 25} dx$, determine the lower and upper limits of integration for θ .

- a) The lower limit is 0. The upper limit is $\frac{\pi}{3}$.
- b) The lower limit is $\frac{\pi}{6}$. The upper limit is $\frac{\pi}{3}$.
- c) The lower limit is $\frac{\pi}{4}$. The upper limit is $\frac{\pi}{3}$.
- d) The lower limit is 0. The upper limit is $\frac{\pi}{6}$.
- e) none of the above

24. What is the area of the largest rectangle that can be inscribed in the region bounded by $y = 3 - x^2$ and the x -axis?

- a) 4
- b) 6
- c) $\frac{3\pi}{2}$
- d) $\sqrt{5}$
- e) none of the above

25. Suppose that you have two linear functions f and g shown below.



Then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ is

- a) 2
- b) does not exist
- c) $\sqrt{2}$
- d) 3
- e) none of the above

26. A ball is thrown straight up from the ground. How high will it go? Assume that g is the absolute value of the gravitational acceleration and v_0 is the initial velocity.

- a) gv_0^2
- b) $\frac{1}{2}g^2 + gv_0$
- c) $\frac{1}{2}v_0 + \frac{1}{2}v_0^2g$
- d) $\frac{1}{2}v_0^2g^{-1}$
- e) none of the above

27. Given $F(x) = \int_0^{x^2} e^{5t-t^2} dt$, find $F'(2)$.

- a) $4e^4$
- b) $-3e^4$
- c) $-3e^4 - 5$
- d) $\frac{1}{5}e^4 - e^{10}$
- e) none of the above

28. Two electrons repel each other with a force inversely proportional to the square of the distance between them, $F = \frac{kq_1q_2}{r^2}$. Suppose one electron is held fixed at the point $(1, 0)$ on the x -axis. Find the work required to move a second electron along the x -axis from the point $(-1, 0)$ to the origin.

- a) $k \frac{q_1q_2}{4}$
- b) $k \frac{q_1q_2}{2}$
- c) kq_1q_2
- d) ∞
- e) none of the above

29. If $f(x) = \sqrt{x} - x + 9$, for $x \geq \frac{1}{2}$, and $g = f^{-1}$, then $g'(9)$ is

- a) -2
- b) $-\frac{5}{6}$
- c) $-\frac{6}{5}$
- d) -1
- e) none of the above

30. Integrate: $\int x \ln(x^2) dx$.

- a) $\frac{x^2 \ln(x^2)}{2} - \frac{x^3}{3} + C$
- b) $\frac{x^2 \ln(x^2)}{2} - \frac{x^2}{2} + C$
- c) $\frac{x^2 \ln(x^2)}{2} + \frac{x^2}{2} + C$
- d) $\frac{x^2 \ln x}{2} - \frac{x}{2} + C$
- e) none of the above

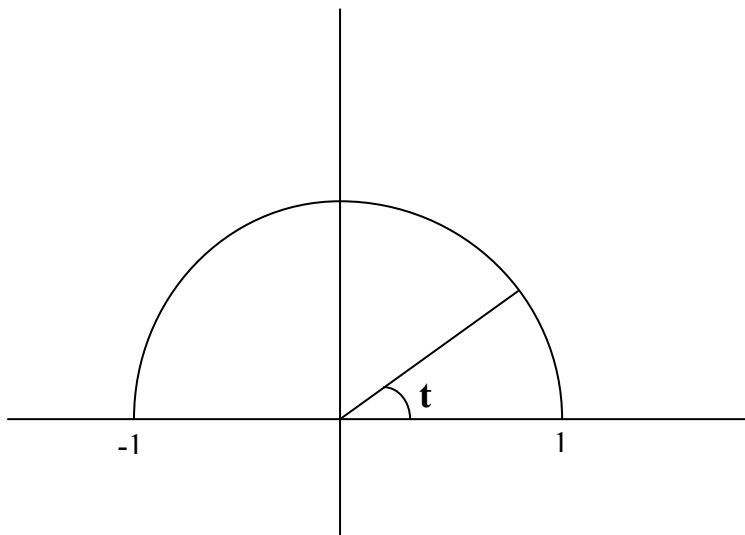
31. Evaluate: $\lim_{x \rightarrow \sqrt[3]{2}} \left(\frac{x^2}{2} - \frac{1}{x} \right)$

- a) $+\infty$
- b) $\frac{2}{3} \cdot \frac{1}{\sqrt[3]{2}}$
- c) 0
- d) $\frac{3}{2} \cdot \sqrt[3]{2} - \frac{1}{\sqrt[3]{2}}$
- e) none of the above

32. If the product function $h(x) = f(x) \cdot g(x)$ is continuous at $x = 0$, then the following must be true about the functions f and g . (Choose just one answer.)

- a) Both functions must be continuous at $x = 0$.
- b) One of them must be continuous at $x = 0$, but not necessarily the other.
- c) Both must be discontinuous at $x = 0$.
- d) They may be continuous or not at $x = 0$, all options are possible.
- e) none of the above

33. One way to compute $\frac{1}{2}$ the area of the unit circle is to integrate $\int_{-1}^1 \sqrt{1-x^2} dx$.



Let t be the angle in radians shown. Then the area of the half circle is

- a) $\int_0^{\pi} -\sin t dt$
- b) $\int_0^{\pi} -\sin^2 t dt$
- c) $\int_0^{\pi} -\cos t dt$
- d) $\int_{\pi}^0 -\sin^2 t dt$
- e) none of the above

34. Evaluate: $\int_{-1}^1 \frac{dx}{x^2 - 6x + 9}$

- a) $\frac{\pi}{4}$
- b) $\frac{\pi}{12}$
- c) $\frac{1}{4}$
- d) $\frac{1}{12}$
- e) none of the above

Reminder

Question 35 will be used again as a tie-breaker, if necessary.

35. Let $f(x)$ be continuous on $[0, 8]$ and twice differentiable on $(0, 8)$. If the average rate of change of $f(x)$ on $[0, 2]$ is 4, the average rate of change of $f(x)$ on $[2, 6]$ is 8, and the average rate of change of $f(x)$ on $[6, 8]$ is 5, which of the following is true?

- a) $f'(x) = 0$ somewhere in $[0, 8]$
- b) $f'(x) = 0$ somewhere in $[0, 8]$ and $f''(x) = 0$ somewhere in $[0, 8]$
- c) $f''(x) = 0$ somewhere in $[0, 8]$
- d) there is not enough information to tell
- e) none of the above

36. Integrate: $\int x^n \cos x \, dx$

- a) $x^n \sin x - n \int x^{n-1} \sin x \, dx$
- b) $-x^n \sin x + n \int x^{n-1} \sin x \, dx$
- c) $x^n \cos x - n \int x^{n-1} \cos x \, dx$
- d) $-x^n \cos x + n \int x^{n-1} \cos x \, dx$
- e) none of the above

37. How many zeros does the function $f(x) = x^4 + 3x + 1$ have in the interval $[-2, -1]$?

- a) no zeros
- b) exactly one zero
- c) exactly two zeros
- d) exactly three zeros
- e) none of the above

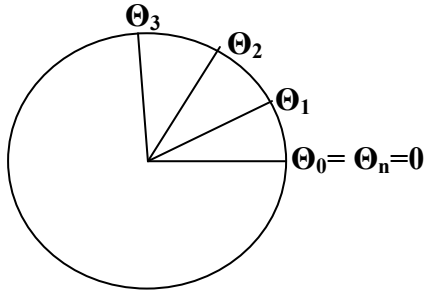
38. Given that $f(n) = \frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \dots + \frac{1}{\sqrt{n}\sqrt{n+n}}$. Find $\lim_{n \rightarrow \infty} f(n)$.

- a) $1 + \frac{1}{e}$
- b) $\sqrt{2} - \frac{1}{e}$
- c) $1 - \frac{1}{e}$
- d) $2\sqrt{2} - 2$
- e) none of the above

39. Evaluate: $\int_2^5 \frac{2}{5x+2} dx$.

- a) 15
- b) $\frac{2}{5} \ln 27 + \frac{2}{5} \ln 12$
- c) $\frac{2}{5} \ln 27 - \frac{2}{5} \ln 12$
- d) $\frac{5}{2} \ln 27 - \frac{5}{2} \ln 12$
- e) none of the above

40. We cut a circular disk of radius r into n circular sectors as shown in the figure, by marking the angles θ_i at which we make the cuts ($\theta_0 = \theta_n$ can be considered to be the angle 0). A circular sector between two angles θ_i and θ_{i+1} has an area $\frac{1}{2}r^2\Delta\theta_i$, where $\Delta\theta_i = \theta_{i+1} - \theta_i$.



We let $A_n = \sum_{i=0}^{n-1} \frac{1}{2} r^2 \Delta\theta_i$. Then the area of the disk, A , is given by:

- a) A_n , independent of how many sectors we cut the disk into
- b) $\lim_{n \rightarrow \infty} A_n$
- c) $\int_0^{2\pi} \frac{1}{2} r^2 d\theta$
- d) all of the above
- e) none of the above