Gainesville State College

Sixteenth Annual Mathematics Tournament

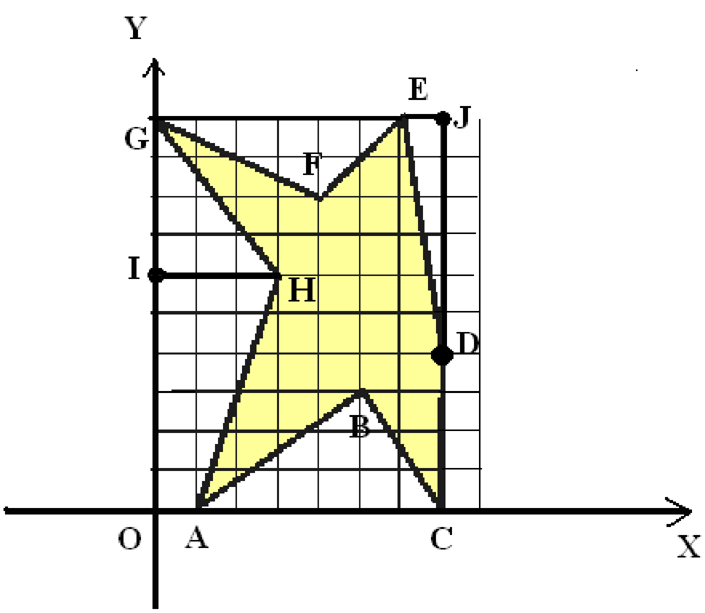
April 10, 2010

Solutions for the Afternoon Team Competition

Round 1

$\frac{\pi}{12}$ is $\frac{1}{12}th$ of π, which is half of the circle, and it is therefore $\frac{1}{24}th$ of a whole circle. Thus, $x = 24$.

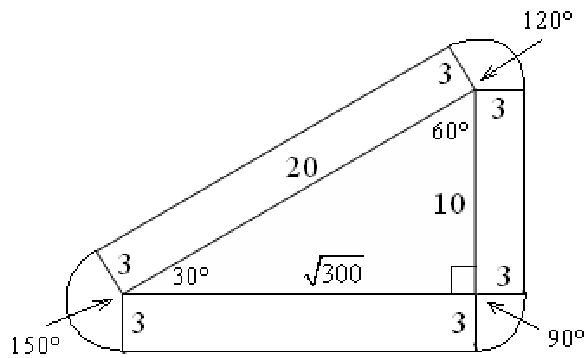
Round 2



The area of the large rectangle OCJG on the picture is 7x10 = 70 sq. units. From this area, we need to subtract the sum of areas of triangles ABC, DEJ, EFG, GHI, and the area of the trapezoid AHIO. We obtain $70-(9+3+6+6+12) = 70-36 = 34\ sq.\,units$.

Round 3

The length of the two sides given shows that the corral is a 30-60-90 degree triangle, with the remaining leg equal to $\sqrt{300}$. The area accessible to the giraffe is made of the triangle, three rectangles, and three partial circles. The circular areas may be calculated by using the proportions of the internal angles to 360 degrees.



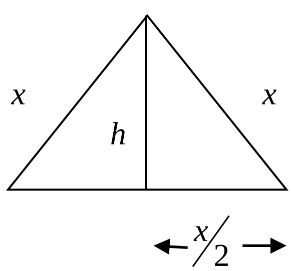
The area is then:

$$A=\left(\frac{1}{2}\cdot 10\cdot\sqrt{300}\right)+(3\cdot 10)+\left(3\cdot\sqrt{300}\right)+(3\cdot 20)+\left(\frac{150}{360}\cdot\pi\cdot 3^2\right)+\left(\frac{120}{360}\cdot\pi\cdot 3^2\right)+\left(\frac{90}{360}\cdot\pi\cdot 3^2\right)=256.838\ m^2$$

Round 4

There are $\dfrac{(867-3)}{2.7}=320$ gaps of 2.7 which means 321 terms altogether.

Round 5



Let one side of the triangle be $x\ cm$ and $h=2\sqrt{3}\ cm$.

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Then $x^2 = \left(\frac{x}{2}\right)^2 + \left(2\sqrt{3}\right)^2$

$$x^2 - \frac{x^2}{4} = 12$$
$$\frac{3}{4}x^2 = 12$$
$$x^2 = 16$$
$$x = 4\ cm$$

So the length of the piece of wire used to construct the equilateral triangle is $3x = 3(4\ cm) = 12\ cm$.

Round 6

During one revolution the wheel revolves 360 degrees. Therefore, during $\frac{11}{6}$ revolutions it revolves $\frac{11}{6} \cdot 360° = 660°$, which is *per minute*. So, $\frac{660°}{60\ sec} = 11°$ *per second.*

Round 7

Using the identity $sin^2 x + cos^2 x = 1$, we can obtain

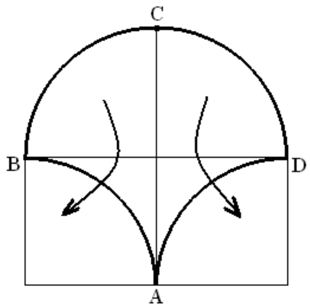
$$\left(2^4\right)^{sin^2 x} + \left(2^4\right)^{1-sin^2 x} = 10$$
$$\left(2^4\right)^{sin^2 x} + \frac{2^4}{\left(2^4\right)^{sin^2 x}} = 10$$

Substituting $t = \left(2^4\right)^{sin^2 x}$, we will obtain the equation $t + \frac{2^4}{t} = 10$. Now multiplying both sides of the equation by *t* and combining all terms on one side, we will get $t^2 - 10t + 16 = 0$. Solving by factorization will give us $(t-8)(t-2) = 0$ and then $t = 8$ *or* $t = 2$. Thus, $\left(2^4\right)^{sin^2 x} = 8 = 2^3$ or $\left(2^4\right)^{sin^2 x} = 2$. Therefore, $4\,sin^2 x = 3$ or $4\,sin^2 x = 1$.

Using these two equations and the fact that sine is positive in the interval $\left[0, \frac{\pi}{2}\right]$, we will obtain $sin\,x = \frac{\sqrt{3}}{2}$ or $sin\,x = \frac{1}{2}$. Thus $x = \frac{\pi}{3}$ or $x = \frac{\pi}{6}$.

Round 8

Divide the semicircle in half and rotate each half to fill the space below the quarter-circles. (See the picture below.) The figure formed is a rectangle of dimension 5 by 10. Thus, the area of a 5x10 rectangle is 50 square units.



Round 9

The orange (O) must be on the second or third place from the left. The banana (B) must be somewhere to the left of the orange. Hence the placement of the banana and the orange may take any of three forms namely BO _ _ , B _ O _ , or _ BO _ . IN each case two ways remain to fill in the open positions with an apple (A) and a pear (P). The total number of ways equals $3 \cdot 2 = 6$. The ways can be listed as follows: BOAP, BPOA, PBOA, BOPA, BAOP, and ABOP.

Round 10

Let my age today be *x*, and your age today be *y*. Then I was as old as you are today $x - y$ years ago. Your age $x - y$ years ago was $y - (x - y)$.
Therefore, we have two equations: $x = 2\left(y - (x - y)\right)$ and $x + y = 63$. Solving them gives $x = 36$ and $y = 27$. So, today I am 36 years old, and you are 27 years old.

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