



Gainesville State College

Eighteenth Annual Mathematics Tournament

April 14, 2012

Solutions for the Afternoon Team Competition

Round 1

There are 3 different choices for Andrea. Since Bob and Andrea can't have vacation during the same month, there are 2 choices for Bob. Dave must also have a different month of vacation than Andrea. Carrie can't have the same month of vacation as Bob or Dave. If Bob and Dave both have vacation the same month, then there are two choices for Carrie's month of vacation. This gives: $3 \times 2 \times 2 = 12$.

If Bob and Dave have different months of vacation, then there is one choice for Carrie's month of vacation.

This gives: $3 \times 2 \times 1 = 6$.

Thus there are $12 + 6 = 18$ total ways to assign summer vacations.

Round 2

Using the remainder theorem, $2^5 - 9 = 23$.

Round 3

The possibilities are:

$$2 \times 3 \times 5 \times 7 \times 11 = 2310$$

$$2 \times 3 \times 5 \times 7 \times 13 = 2730$$

$$2 \times 3 \times 5 \times 7 \times 17 = 3570$$

$$2 \times 3 \times 5 \times 7 \times 19 = 3990$$

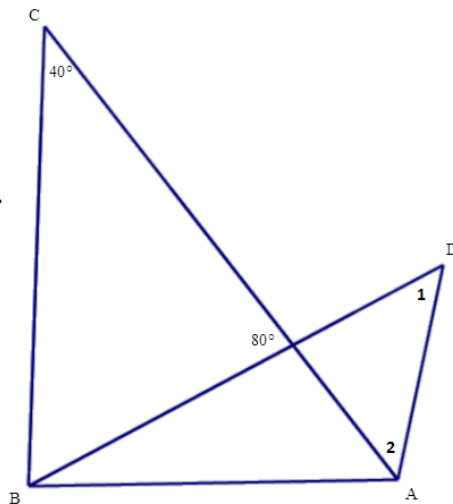
$$2 \times 3 \times 5 \times 7 \times 23 = 4830$$

$$2 \times 3 \times 5 \times 11 \times 13 = 4290$$

So, there are 6 possible numbers.

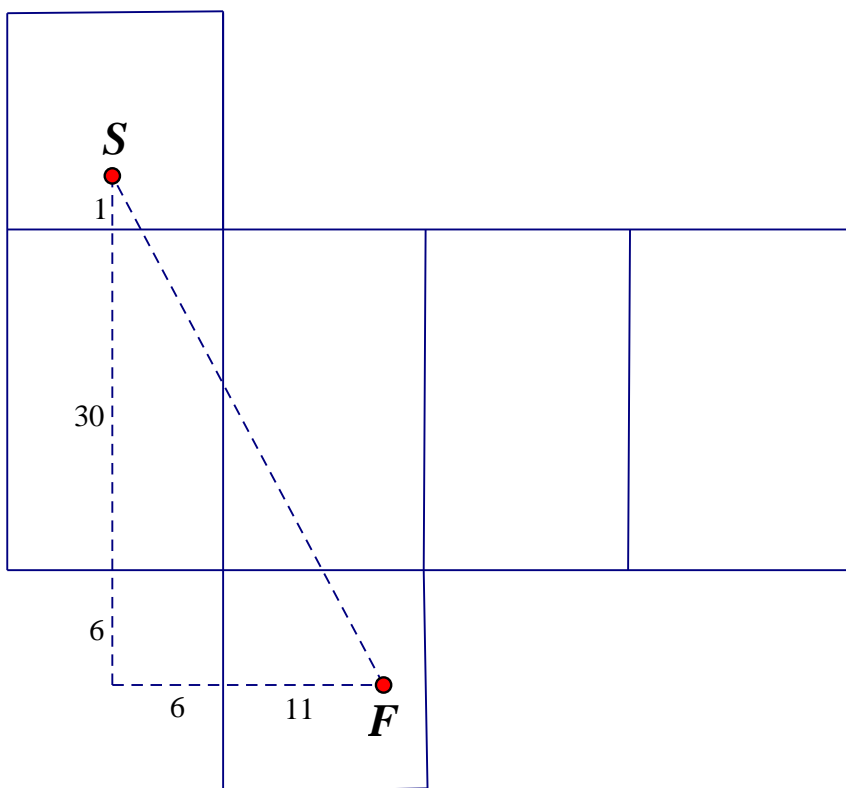
Round 4

The angle opposite the 80° angle is also 80° . Let $\angle 1 = \angle 2 = x$.
 Then $80^\circ + 2x = 180^\circ$ and $x = 50^\circ$ so, $\angle 1 = 50^\circ$.
 So, $\angle DBA = 180^\circ - \angle DAB - \angle 1 = 180^\circ - 103^\circ - 50^\circ = 27^\circ$.



Round 5

Consider the diagram below of a net of the room, with S indicating the initial position of the spider and F indicating the initial location of the fly.



Finding the distance from S to F is equivalent to finding the length of the hypotenuse of a right triangle whose legs are 37 feet and 17 feet long. We have:

$$37^2 + 17^2 = d^2$$

$$1369 + 289 = d^2$$

$$1658 = d^2$$

$$d \approx 40.7 \text{ feet}$$

Round 6

The sequence repeats after every 7 terms. Thus, $T_1 = T_{1+7 \cdot 287} = T_{2010}$.

$$T_{2011} = 0$$

$$T_{2012} = 0$$

$$T_{2013} = 1$$

$$T_{2014} = 2$$

$$(T_{2014} - T_{2012}) : (T_{2013} - T_{2011})$$

$$(2 - 0) : (1 - 0)$$

So the ratio is: 2:1.

Round 7

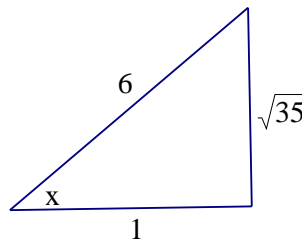
We have the double-angle formula: $\cos(2x) = 2\cos^2 x - 1$.

Substitute into the equation to get: $3(2\cos^2 x - 1) + 17\cos x = 0$.

This gives: $6\cos^2 x + 17\cos x - 3 = 0$. Factoring gives: $(\cos x + 3)(6\cos x - 1) = 0$.

So $\cos x = -3$ or $\cos x = \frac{1}{6}$. Since $\cos x$ can't be equal to -3, then $\cos x = \frac{1}{6}$.

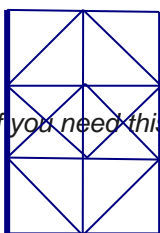
If $\cos x = \frac{1}{6}$, then consider a triangle such as:



Then either $\tan x = \sqrt{35}$ or $\tan x = -\sqrt{35}$, so $\tan^2 x = 35$.

Round 8

Consider the following drawing that shows how the points could be connected to form rectangles.



There are six 1 by 1 squares, three 1 by 2 rectangles, two 2 by 2 squares, two 3 by 1 rectangles, four 2 by 1 rectangles, and one 3 by 2 rectangle.

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This adds up to 18. There are also the following two squares with vertices: (0, 1) (1, 2) (2, 1) (1, 0) and (0, 2) (1, 3) (2, 2) (1, 1) So there are a total of 20 rectangles.

Round 9

Let S be the swimmer's speed in still water and let C be the speed of the current.

Suppose the swimmer swims against the current at a rate of $S - C$ for 10 minutes. Then his distance traveled during that time would be $(S - C)10$.

Then suppose the swimmer swims downstream at a rate of $S + C$ for x minutes. Then his distance traveled during that time would be $(S + C)x$.

Since the second distance is 1000 yards longer, $(S + C)x - (S - C)10 = 1000$

During this time, the hat has floated 1000 yards at a rate of C . So $C(10 + x) = 1000$

So there are two equations:

$Sx + Cx - 10S + 10C = 1000$ or if you rearrange the terms $Sx - 10S + 10C + Cx = 1000$ and

$$10C + Cx = 1000$$

Substitute 1000 into the first equation for $10C + Cx$

$$Sx - 10S + 1000 = 1000$$

$$Sx = 10S$$

$$x = 10$$

Substitute into the equation above

$$10C + C(10) = 1000$$

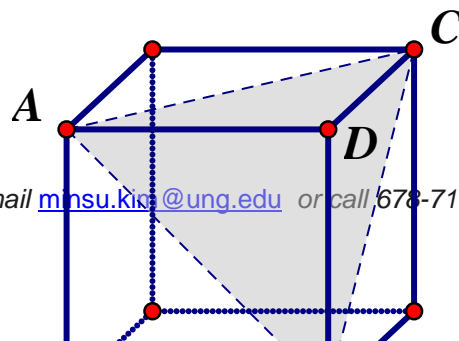
$$20C = 1000$$

$C = 50$ yards per minute The speed of the current is 50 yards per minute.

Round 10

The pyramid can be viewed in such a way that the base is a right triangle that is half of one of the cube's sides.

$$V = (1/3)(\text{area of base})(\text{height})$$



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$$972 = \frac{1}{3} \left(\frac{1}{2} s^2 \right) (s)$$

$$972 = \frac{1}{6} s^3$$

$$5832 = s^3$$

$$s = 18 \text{ meters}$$