University of North Georgia  
Mathematics Tournament  
April 7, 2018

Solutions for the Afternoon Team Competition

Round 1

From the figure below: \( \tan \alpha = \frac{6}{x} = \frac{15 - x}{6} \). Simplifying gives \( 36 = 15x - x^2 \) and then

\[ x^2 - 15x + 36 = 0. \]

Solving gives \( x = 3 \) or \( x = 12 \). So \( \tan \alpha = \frac{6}{12} = \frac{1}{2} \) or \( \tan \alpha = \frac{6}{3} = 2 \).

Round 2

Use as many 12 lb. bags as possible, and let the gap be some number that will be filled up by a combination of 18 lb. and 22 lb. bags. 1000 divided by 12 gives 83 1/3, so try 80 and get 80*12 = 960 leaving a gap of 40 bags. So you need 80 12 lb. bags, 1 18 lb. bag, and 1 22 lb. bag for a total of 82 bags.

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Round 3

Since it is stipulated that $0^\circ \leq A \leq B \leq 180^\circ$, if you view the equations as resulting from two unit vectors, one at angle $A$ from the positive $x$-axis and the other unit vector at angle $B$, $\cos A + \cos B = 0$ implies that $\sin A = \sin B$. So from $\sin A + \sin B = 0.5$ and $\sin A = \sin B$, we have $\sin A = 0.25$. Solving gives $A = 14.47751219^\circ$. Angle $B = 180 - A$, so the difference $B - A = 180 - 2A = 180 - 2(14.47751219) = 151.0449756^\circ = 151^\circ$.

Round 4

$P(\text{snow}) = P(\text{snow} \cup \text{rain}) - P(\text{rain}) + P(\text{snow} \cap \text{rain}) = 0.8 - 0.4 + 0.1 = 0.5$ or $50\%$ chance

Round 5

Let $x =$ the number of oranges at the beginning. We have the equation:

$$\frac{x - 1}{2} - \frac{1}{2} = 24.$$  Multiplying both sides of the equation by 2 gives

$$\frac{x - 1}{2} - \frac{1}{2} - 1 = 48\text{ and then } \frac{x - 1}{2} - \frac{3}{2} = 48.$$  Multiplying both sides of the equation by 2 again gives

$$\frac{x - 1}{2} - \frac{3}{2} = 96.$$  Solving gives $x = 199$.

Round 6

We have $f(x) = \begin{cases} -2x + 4035 & \text{for } x < 2017 \\ 1 & \text{for } 2017 \leq x \leq 2018 \\ 2x - 4035 & \text{for } x > 2018 \end{cases}$

The first function is decreasing, the second function is constant, and the third is increasing. Thus the minimum of $f(x)$ is when $2017 \leq x \leq 2018$ and it is equal to 1.
Round 7

The integer coordinates that satisfy the equation are \((\pm 3, \pm 4), (\pm 4, \pm 3), (0, \pm 5), \text{ and } (\pm 5, 0)\).

To have the greatest possible ratio \(\frac{AB}{CD}\), we want to maximize \(AB\) and minimize \(CD\). Since they are both irrational, they also have to be the square root of something. The greatest value of \(AB\) happens when \(A\) and \(B\) are almost across from each other and are in opposite quadrants. So \(A\) could be \((-4, 3)\), \(B\) could be \((3, -4)\), and then \(AB = \sqrt{98}\). The least value of \(CD\) happens when \(C\) and \(D\) are in the same quadrant and very close to each other. So \(C\) could be \((3, 4)\), \(D\) could be \((4, 3)\), and then \(CD = \sqrt{2}\). Thus,

\[
\frac{AB}{CD} = \frac{\sqrt{98}}{\sqrt{2}} = \sqrt{49} = 7.
\]

Round 8

When the graph of the left side is intersected by the line \(y = m\), we can get two, three, four, or no intersection points. Exactly three solutions are obtained when the line \(y = m\) touches the vertex of the parabola \(y = -(x^2 + 4x - 5)\). See the picture below. The vertex of the parabola \(y = x^2 + 4x - 5\) is at the point \((-2, -9)\), so the vertex of the parabola \(y = -(x^2 + 4x - 5)\) is at the point \((-2, 9)\). So the line has the equation \(y = 9\), therefore \(m = 9\).

Round 9

Let \(a_1\) be the first term and \(r\) be the constant difference between two consecutive terms. Then

\[
a_n = a_1 + (n-1)r.
\]

We have the system of equations \[
\begin{align*}
-7 &= a_1 + (3-1)r \\
56 &= a_1 + (12-1)r
\end{align*}
\]

\(a_1 = -21\) and \(r = 7\). Then we have \(28 = -21 + (n-1)\cdot 7\), so \(n = 8\).

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Round 10

Since the sum of the squares of the sides of a right triangle is 578, we have \( a^2 + b^2 + c^2 = 578 \). Substituting \( a^2 + b^2 \) for \( c^2 \) gives \( a^2 + b^2 + \left( a^2 + b^2 \right) = 578 \). Combining terms gives \( 2a^2 + 2b^2 = 578 \) and
\[
2a^2 + 2b^2 = 289 \quad (1).
\]

Since the perimeter of the right triangle is 40, we have \( a + b + c = 40 \). Substituting for \( c \) gives
\[
a + b + \sqrt{a^2 + b^2} = 40 \quad (2).
\]

Using (1) above, \( \sqrt{a^2 + b^2} = 17 \). Substituting into (2) gives \( a + b + 17 = 40 \) and then \( a + b = 23 \). Solving for \( b \) and substituting into (1) gives \( a^2 + (23 - a)^2 = 289 \). Simplifying this equation gives
\[
a^2 + 529 - 46a + a^2 = 289 \quad \text{and then} \quad 2a^2 - 46a + 240 = 0.
\]
Solving this equation gives \( a = 8 \) or \( a = 15 \). Then \( b = 23 - a = 23 - 8 = 15 \) or \( b = 23 - a = 23 - 15 = 8 \). So the length of the smallest side of the right triangle is 8.