University of North Georgia

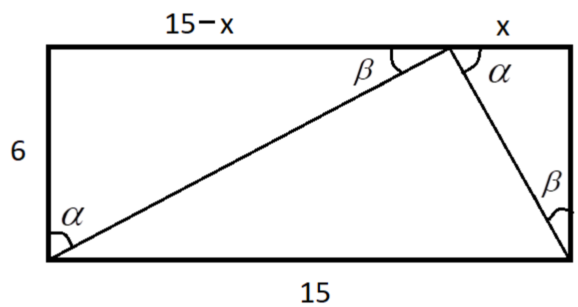
Mathematics Tournament

April 7, 2018

Solutions for the Afternoon Team Competition

Round 1

From the figure below: $\tan\alpha = \frac{6}{x} = \frac{15-x}{6}$. Simplifying gives $36 = 15x - x^2$ and then $x^2 - 15x + 36 = 0$. Solving gives $x = 3$ or $x = 12$. So $\tan\alpha = \frac{6}{x} = \frac{6}{12} = \frac{1}{2}$ or $\tan\alpha = \frac{6}{x} = \frac{6}{3} = 2$.



Round 2

Use as many 12 lb. bags as possible, and let the gap be some number that will be filled up by a combination of 18 lb. and 22 lb. bags. 1000 divided by 12 gives 83 1/3, so try 80 and get $80*12 = 960$ leaving a gap of 40 bags. So you need 80 12 lb. bags, 1 18 lb. bag, and 1 22 lb. bag for a total of 82 bags.

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Round 3

Since it is stipulated that $0° \le A \le B \le 180°$, if you view the equations as resulting from two unit vectors, one at angle A from the positive x-axis and the other unit vector at angle B, $\cos A + \cos B = 0$ implies that $\sin A = \sin B$. So from $\sin A + \sin B = 0.5$ and $\sin A = \sin B$, we have $\sin A = 0.25$. Solving gives $A = 14.47751219°$. Angle $B = 180 - A$, so the difference $B - A = 180 - 2A = 180 - 2(14.47751219) = 151.0449756° = 151°$.

Round 4

$$P(snow) = P(snow \cup rain) - P(rain) + P(snow \cap rain) = 0.8 - 0.4 + 0.1 = 0.5 \text{ or } 50\% \text{ chance}$$

Round 5

Let x = the number of oranges at the beginning. We have the equation:

$\dfrac{\dfrac{\frac{x}{2}-\frac{1}{2}}{2}-\dfrac{1}{2}}{2}-\dfrac{1}{2}=24$. Multiplying both sides of the equation by 2 gives $\dfrac{\frac{x}{2}-\frac{1}{2}}{2}-\dfrac{1}{2}-1=48$ and then $\dfrac{\frac{x}{2}-\frac{1}{2}}{2}-\dfrac{3}{2}=48$. Multiplying both sides of the equation by 2 again gives $\dfrac{x}{2}-\dfrac{1}{2}-3=96$. Solving gives $x=199$.

Round 6

We have $f(x) = \begin{cases} -2x + 4035 & \text{for } x < 2017 \\ 1 & \text{for } 2017 \le x \le 2018 \\ 2x - 4035 & \text{for } x > 2018 \end{cases}$

The first function is decreasing, the second function is constant, and the third is increasing. Thus the minimum of $f(x)$ is when $2017 \le x \le 2018$ and it is equal to 1.

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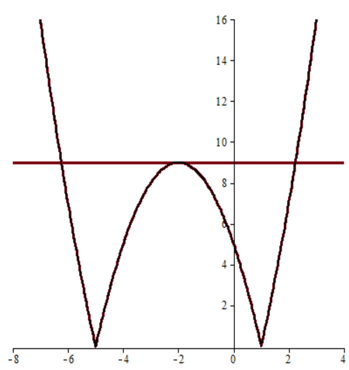
Round 7

The integer coordinates that satisfy the equation are $(\pm 3, \pm 4)$, $(\pm 4, \pm 3)$, $(0, \pm 5)$, and $(\pm 5, 0)$.

To have the greatest possible ratio $\frac{AB}{CD}$, we want to maximize *AB* and minimize *CD*. Since they are both irrational, they also have to be the square root of something. The greatest value of *AB* happens when *A* and *B* are almost across from each other and are in opposite quadrants. So A could be $(-4, 3)$, B could be $(3, -4)$, and then $AB = \sqrt{98}$. The least value of *CD* happens when *C* and *D* are in the same quadrant and
very close to each other. So C could be $(3, 4)$, D could be $(4, 3)$, and then $CD = \sqrt{2}$. Thus,
$\frac{AB}{CD} = \frac{\sqrt{98}}{\sqrt{2}} = \sqrt{49} = 7$.

Round 8

When the graph of the left side is intersected by the line $y = m$, we can get two, three, four, or no intersection points. Exactly three solutions are obtained when the line $y = m$ touches the vertex of the parabola $y = -(x^2 + 4x - 5)$. See the picture below. The vertex of the parabola $y = x^2 + 4x - 5$ is at the point $(-2, -9)$, so the vertex of the parabola $y = -(x^2 + 4x - 5)$ is at the point $(-2, 9)$. So the line has the equation $y = 9$, therefore $m = 9$.



Round 9

Let a_1 be the first term and *r* be the constant difference between two consecutive terms. Then $a_n = a_1 + (n-1)r$. We have the system of equations $\begin{cases} -7 = a_1 + (3-1)r \\ 56 = a_1 + (12-1)r \end{cases}$. Solving this system gives $a_1 = -21$ and $r = 7$. Then we have $28 = -21 + (n-1) \cdot 7$, so $n = 8$.

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Round 10

Since the sum of the squares of the sides of a right triangle is 578, we have $a^2 + b^2 + c^2 = 578$.
Substituting $a^2 + b^2$ for c^2 gives $a^2 + b^2 + \left(a^2 + b^2\right) = 578$. Combining terms gives $2a^2 + 2b^2 = 578$ and $a^2 + b^2 = 289$ (1).
Since the perimeter of the right triangle is 40, we have $a + b + c = 40$. Substituting for *c* gives $a + b + \sqrt{a^2 + b^2} = 40$ (2).
Using (1) above, $\sqrt{a^2 + b^2} = 17$. Substituting into (2) gives $a + b + 17 = 40$ and then $a + b = 23$.
Solving for *b* and substituting into (1) gives $a^2 + (23 - a)^2 = 289$. Simplifying this equation gives $a^2 + 529 - 46a + a^2 = 289$ and then $2a^2 - 46a + 240 = 0$. Solving this equation gives $a = 8$ or $a = 15$.
Then $b = 23 - a = 23 - 8 = 15$ or $b = 23 - a = 23 - 15 = 8$. So the length of the smallest side of the right triangle is 8.