

CURCA Proposal Cover Sheet

Title of Project: Mathematical Modeling and Simulation of Strategy-based Games

Faculty Members:

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Mathematics & Computer Science

Projected End Date: April 30, 2011 **Mathematical Modeling and Simulation of Strategy-Based Games**

Project Description:

Several games employ many different kinds of individual and team strategies. Two such games are of interest for this project. The first is the recently developed commercial game Gobblet Gobblers, which is a more evolved version of Tic-Tac-Toe with increased complexity. Like Tic-Tac-Toe, the object of the game is to be the first to line up three like-colored pieces. However, in this game, pieces come in three different sizes, and larger pieces may be played to cover opponents' smaller pieces. Players may also relocate any of their own pieces which are already in play.

The second is a game of our own invention, derived from military-style laser tag. In this game, a player is eliminated only after all still-active players on the opposing team have 'tagged' that player. One interesting implication of this rule is that a player can inadvertently remove himself from play by tagging the only member of the opposing team who has not yet tagged him. Additionally, this game has several parameters that are open to variation; such parameters include the sizes of the teams and the likelihood of a "hit" each time a player fires.

Research participants will use graph theory and probability to build mathematical models to analyze these games and seek optimal team and individual strategies for each game. Furthermore, student researchers will work with faculty to develop computer simulations for the games. These simulations will be used to compute both theoretical and empirical probabilities associated with the game and thereby determine optimal strategies. To develop the simulations, the student researchers will learn to use a cutting-edge parallel-processing language called CUDA in a Graphics Processing Unit (GPU) programming environment.

Significance:

The significance of this project is three-fold, involving 1) applications of mathematical modeling across many disciplines; 2) the rapidly growing field of graph theoretic models in mathematics; and 3) the evolving field of parallel computer simulations in GPU programming environments.

First, mathematical modeling has many inter-disciplinary applications. Many facets of games model real-world phenomena that can be found in fields such as business, psychology, sociology, computer networking, and many others. If a research question from such a discipline were to be re-stated in the language of a mathematical game, like one of the ones we describe, then mathematical and computer analysis could yield the information or results that the original researcher sought. In particular, the research results can specify the probability of obtaining a desirable or acceptable outcome, given a particular configuration of starting conditions and assumptions. These results can then be extended to discover optimal strategies or courses of action for a given set of conditions. This project seeks to contribute to the body of knowledge that allows such mathematical modeling of phenomena in a variety of academic fields.

Second, graph theoretic modeling is a burgeoning and rapidly developing field of mathematics. The methods that we develop for studying these games may later be applied to other games, or by extension, even to real-world phenomena. In addition, it appears that these two games have never been mathematically analyzed using graph theoretic models. Both games (described previously) have tremendous potential as viable applications of graph theoretic models.

Since one of the games is a relatively new yet commercially available game that is growing in popularity, there may be widespread interest among mathematicians and game theorists in understanding the strategic principles underlying the game. The other game has potential applications in military and other strategic settings and may be of interest in such fields as military science, law enforcement, and political science. In both cases, graph theoretic models provide an appropriate and relevant mechanism to analyze the games. This project seeks to leverage developments in the area of graph theoretic models, as well as to make further contributions in this evolving area of mathematics research.

Third, parallel processing, and in particular parallel programming in a GPU environment, is one of the most significant current developments in the the field of computer science. This area of computing, known as General Purpose GPU (GPGPU) programming, is advancing to the forefront of the computing industry. One of the most pervasive uses for this kind of programming is to conduct simulations, because a simulation is computationally intensive, entailing many different processing paths for which results need to be determined. GPU environments allow the programmer to compute many outcomes simultaneously, thereby maximizing the number of different simulation results that can be obtained in a limited amount of time. This allows far more complex simulation scenarios than were previously possible. Finally, graph theoretic applications are particularly well suited to computer-programmed representations and simulations. Graph theory and computing are often jointly researched because data structures in computer science provide a very useful way to represent the elements and structure of the graphs.

Thus, the proposed project is highly inter-disciplinary, with applications relevant to many fields, and with the planned research having a direct impact on significant lines of inquiry in both mathematics and computer science.

Goals and Products:

We will model these games to determine probabilities of different outcomes given specified sets of conditions then use the data to determine optimal strategies.

The deliverables for this project include: code for computer simulations, mathematical analysis of the outcomes of each game and computations of theoretical/empirical probabilities of victory under various conditions. In addition, each of the students will write a brief paper summarizing their work and present their work at a regional, national or international conference. The paper will be suitable for submission to an undergraduate research journal, such as *Involve*. Our goal is to have the students present at the Southeastern International Conference on Combinatorics, Graph Theory and Computing (SICCGTC), held annually in Florida, usually during March. This academic meeting is the most suitable venue because it is among the most prestigious conferences for this type of research and is historically very welcoming to students. We will apply for funding from the VPAA's office to support the student's travel and will seek funding from our Dean's office and our Department travel funds to support the travel of the accompanying faculty member.

Plan for Involvement of Undergraduates:

Students will be the primary researchers, with Drs. Bailey, Holliday, and Spence acting as research mentors. We will establish a schedule of regular meetings with the students to monitor progress, discuss the research, provide direction and assist with preparation of formal results for publication/presentation. There will be four students involved in our project; we have already recruited students.

Budget Summary:

A summary of the proposed budget is given below. See the next section for a detailed justification of each line item in the budget.

	Individual Cost	Line Item Total
Student Stipends	\$400 each (x 4)	\$1600.00
Materials & Supplies	\$275	\$267.00
Equipment	\$1125	\$1125.00
Total	-----	\$2992.00

Detailed Budget Justification:

Student Stipends: Each of the 4 participating students will be paid a total of \$400 for his/her participation throughout the 2011-2012 academic year. The student stipends will be paid out in two equal installments, one at the end of each semester. These stipends will compensate the students for their time and provide additional motivation to see the project through (besides their natural love of learning and problem-solving). Although we do not anticipate this, if a student does not remain involved in the project through the second semester, that student will not receive the second installment of the stipend.

Materials and Supplies: The project team members will require copies of the game Gobblet Gobblers in order to allow students working independently to become familiar enough with the game to be able to model it mathematically and develop the computer code to simulate it. Both instructors and students will also need reference books, particularly those on the CUDA programming language and on parallel programming algorithms and strategies, which will be needed to code the simulations. Enough copies of each are needed for a team of seven (3 instructors and 4 students) to have reasonable access to these resources throughout the course of the research project. These supplies and their cost are detailed below.

Item	Retail Price	Quantity	Cost
Game: Gobblet Gobblers	20.00	3	60.00
Book: Sanders & Kandrot, CUDA By Example	26.50	3	73.50
Book: Kirk & Hwu, Programming Massively Parallel Processors	44.50	3	133.50
Total	----- ---	----- -	267.00

Equipment: The CUDA programming language requires a platform with an Nvidia GPU card and development environment. To support the GPU card for use in parallel programming, the platform requires

certain hardware, memory, and power supply specifications beyond those found in computers available in the Math/CS Department or in campus computer labs. An economical platform that will meet the needs of the project has been specified with the components below. These will be combined with a mouse and a monitor available in the Math/CS Department and already secured for the project.

Component	Cost
Motherboard - ASUS P8H61-M LE/CSM	75.00
CPU - Intel Core i7-2600	300.00
GPU Card - EVGA 570 GTX 1.25 GB	325.00
Memory - 2 x Kingston DDR3 1333 4GB	50.00
Hard Drive - WD5002AALX 500GB	70.00
Power Supply - Corsair TX650 V2 650Watt	100.00
Keyboard - Logitech K200	20.00
CD/DVD - LG GH24LS70	25.00
OS/Software Configuration	160.00
Windows 7 Professional 64Bit OEM	
Visual Studio 2008 Compiler	
Total	1125.00

This computer will be placed in the NOC 211 lab where our research students will have access. This lab is locked at night and whenever the department is closed for business.

Timeline:

Early October	Meet with students to introduce the game(s). Set up weekly meetings.
October- Early February	Regular weekly meetings with the student researchers and faculty mentor.
Early-February- Early March	Students prepare to present results; continue to meet with faculty for guidance.
Mid-March	Students present at SICCGTC.
March-April	Students project summaries suitable for publication;

continue to meet with faculty for guidance.